BULLETIN OF THE SCHOOL OF EDUCATION INDIANA UNIVERSITY

Vol. XXI | January, 1945 | No. 1

ONE HUNDRED FIFTY YEARS OF ARITHMETIC TEXTBOOKS

Bureau of Cooperative Research and Field Service
School of Education
Indiana University, Bloomington, Indiana

HENRY LESTER SMITH, Director
MERRILL T. EATON, Assistant Director

Price, 50 cents, postpaid
For sale by the Indiana University Bookstore,
Bloomington, Indiana

This bulletin is indexed by title in the following:
Bibliographic Index
The current literature section of the
Wilson Education Index

Entered as second-class mail matter September 30, 1924, at the post office at Bloomington, Indiana, under the Act of August 24, 1912. Published in January, March, May, July, September, and November of each year, from the University Office, Bloomington, Indiana.

Price, 50 cents, postpaid
For sale by the Indiana University Bookstore,
Bloomington, Indiana

This bulletin is indexed by title in the following:
Bibliographia Index
The current literature section of the
Wilson Education Index

Entered as second-class mail matter September 30, 1924, at the post office at Bloomington, Indiana, under the Act of August 24, 1912. Published in January, March, May, July, September, and November of each year, from the University Office, Bloomington, Indiana.

BULLETIN OF THE SCHOOL OF EDUCATION
INDIANA UNIVERSITY

ONE HUNDRED FIFTY YEARS OF ARITHMETIC TEXTBOOKS

By

HENRY LESTER SMITH
MERRILL T. EATON
KATHLEEN DUGDALE

Copyright, 1945, by the Bureau of Cooperative Research and Field Service, Indiana University

Published by the
BUREAU OF COOPERATIVE RESEARCH AND FIELD SERVICE
Indiana University

Acknowledgment

THE authors wish to express their appreciation for the assistance given them by various individuals and by textbook publishers in the development of this study.

Since this is the sixth and last of a series of analyses of arithmetic textbooks, a study carried on by the Bureau of Cooperative Research and Field Service, it is in some ways a summary of findings published in the earlier bulletins in the series. The authors therefore wish to express their thanks to those who assisted with the five previous analyses as well as to Mrs. Laura Green, who collected much of the additional material on which this final study is based.

Several individuals have read the manuscript critically and have made helpful suggestions. Appreciation for such service is extended to Robert Stewart McElhinney, assistant in the Bureau of Cooperative Research and Field Service; Velorus Martz, Professor of Elementary Education; Philip Peak, Instructor in Education and Critic Teacher in Mathematics in the University School; and Mrs. Mabel C. Culmer, Instructor in Education and Reading Clinician, all of Indiana University.

The generosity of publishers in giving permission to quote or to photograph material from their textbooks has been an important factor in making this study possible. Publishers who have cooperated in this respect are: American Book Company; Ginn and Company; D. C. Heath and Company; Houghton Mifflin Company; Iroquois Publishing Company, Inc.; Johnson Publishing Company; Lyons and Carnahan; The Macmillan Company; Benj. H. Sanborn and Company; Scott, Foresman and Company; Silver Burdett Company; and The John C. Winston Company.

It is hoped that persons reading this study will find it both interesting and informational.

H.L.S.
M.T.E.
K.D.

TABLE OF CONTENTS

PAGE

ONE HUNDRED FIFTY YEARS OF ARITHMETIC TEXTBOOKS

INTRODUCTION 1
Books Analyzed in This Study 2

PURPOSES 8

PHYSICAL MAKE-UP 14
Size of Books 14
Covers of Books 14
Number of Volumes in Texts 18
Titles of Texts 20
Appearance of Page 22
Interest of Authors and Publishers in Physical Make-up of Book 25

CONTENT 27
Phases of Arithmetic Included 27
Ten phases of arithmetic given the most space 28
Other phases of arithmetic included 30
Order in which phases were taken up 32
Types of Business Transactions Included 33
Types of business transactions included in at least two thirds of the texts 33
Other business transactions included 36
Arithmetic of Daily-Life Activities 37
Forms of Presentation 38
Principal means of presenting arithmetic 39
Other forms of presentation 42
Non-Mathematical Content 46
Title page 46
Preface 46
Table of contents 46
Index 48
Appendix 48
Recommendations 48
Variety of Content in Each Text 50

METHOD 53
Role of the Teacher 54
Methods of Presenting Material 56
Deductive method 56
Inductive method 57
Topical method 58
Grube method 59

TABLE OF CONTENTS (Cont.)

Spiral method 60
Activity method 61
Important Teaching Theories and Techniques Prominent in Texts During the Nineteenth Century 62
Object method 62
Inclusion of answers 65
Use of analysis 66
Theory of mental discipline 68
Oral and written arithmetic 69
Number taught through measurement 70
Important New Teaching Theories and Techniques 70
Illustrations 70
Units of instruction 72
Review material 77
Drill material 78
Tests 81
Providing for individual differences 81
The personal element 83
Adjustment to Grade Level Needs 85
Simplification of topics 85
Simplification of rules and definitions 87
Simplification of tables of measures 92
Simplification of vocabulary 95
Division of material according to grade level 95
Lesser Techniques 96
Explanation of symbols 97
Different methods of performing the same process 97
Short cuts 99
Poetry 99
Rules of proof 100
Use of blackboard 100
Use of analogies 100
Assistance in comprehension 101
Motivation 101
Rivalry and desire for approval 101
Creation of interest 103
Development of a feeling of need for arithmetic 105
WRITTEN PROBLEMS 107
Written Problems Common to All Textbooks 107
Practical business problems 107
Problems involving measurement 110
Problems based on historical or scientific facts 111
Problems concerning children's daily-life activities 112
Problems Typical of Old Arithmetics 113
Questions on rules and definitions 115
Questions on a certain topic 116
Problems in verse 117
Puzzle problems 119

TABLE OF CONTENTS (Cont.)

Questions at foot of page 123
Problems involving processes now dropped from texts 124
Ridiculous Problems 127
Problems Typical of Recent Arithmetics 131
Changes in subject matter 132
Changes in form 132
Subject Matter of Written Problems 134
Classification of nouns used 135
Comparison of subject matter of early and late problems .. 137
SUMMARY 144
List of Bulletins in the Field of Education, Indiana University 150

LIST OF TABLES

TABLE		PAGE
I.	Phases of Arithmetic Given an Average of 3 Per Cent or More of Space in Textbooks of at Least Four of the Five 30-Year Periods	28
II.	Forms of Presentation Used in All Arithmetic Textbooks	39
III.	Number of Different Concrete Nouns Used in Each Subject Field and Number of Times Such Words Were Used	136
IV.	Two Hundred Examples of Words Showing a Definite Tendency to Drop Out or to Increase in Use	138

LIST OF FIGURES

FIGURE		PAGE
1.	First page of preface of *Cocker's Arithmetick* (1677)	viii
2.	Typical covers of textbooks of various periods	15
3.	Pages from representative arithmetics, showing the comparative size of page, size and kind of type, and general attractiveness of the page	23
4.	Examples of tables found in arithmetics published before 1830	42
5.	Form in which mathematical symbols were explained in early textbooks	45
6.	Graph of certain kinds of arithmetic content, showing in which textbooks each is included	51
7.	Tables of contents of texts of various periods, showing the arrangement and organization of topics	59
8.	Various methods of teaching numeration	64
9.	An example of the use of illustrations for instructional purposes	71
10.	Typical units of instruction	75
11.	One method used for making drill on the fundamentals more attractive	80
12.	Provision for individual differences	83
13.	Page taken from a sum book written in 1840	102
14.	Typical pages of written problems found in arithmetic textbooks published before 1850	114

(1:A5)

Mr. *Edward Cocker's* PROEME or PREFACE.

BY the ſacred Influence of Divine Providence, I have been Inſtrumental to the benefit of many, by vertue of thoſe uſeful Arts, Writing *and* Engraving: *And do now with the ſame wonted alacrity caſt this my Arithmetical Mite into the Publick Treaſury, beſeeching the Almighty to grant the like Bleſſing to theſe as to my former Labours.*

Seven Sciences ſupremely excellent,
Are the chief Stars in Wiſdoms Firmament,
Whereof Arithmetick is one, whoſe worth
The Beams of Profit and Delight ſhines forth;
This crowns the reſt; this makes Man's mind compleat;
This treats of Numbers, and of this we treat.

I have been often deſired by my intimate Friends to publiſh ſomthing on this Subject; who in a pleaſing Freedom have ſignified to me that they expected it would be extraordinary. How far I have anſwered their Expectation, I know not; but this I know, that I have deſigned this Work not extraordinary abſtruſe or profou-

(full size)

Fig. 1. First page of preface of *Cocker's Arithmetick* (1677)

ONE HUNDRED FIFTY YEARS OF ARITHMETIC TEXTBOOKS

Chapter I
Introduction

THE study of arithmetic is looked upon today as so fundamental a part of the school curriculum of our country that it is hard to realize the subject has not always held such a prominent position. True, it was among the basic three subjects—"reading, 'riting, and 'rithmetic"—from which our broad, present-day curriculum has developed, but it was the last of these three to be offered. Not until about the time of the Revolutionary War was it included as a required school subject, while reading and writing had been taught in the schools of Massachusetts, a leader in many phases of the early development of our public schools, as early as the middle of the seventeenth century.

With the inclusion of arithmetic in the curriculum, the need for arithmetic texts for school children arose. Before this time books on the subject were, for the most part, treatises on the more complicated phases of mathematical science. These were usually written for persons in business who were in need of instruction or for those who studied the subject from a purely scientific or philosophical point of view.

The earliest arithmetic in the English language was *The Grounds of Artes*,[1] written by Robert Recorde in 1540, only 400 years ago. It was much later, however, when the first arithmetic was published in what is now the United States. William Bradford, the first New York printer and a native author, published this text in 1705.[2]

In these early days there were but few persons interested in mathematics books and still fewer capable of writing them and therefore, until the demand was increased through the teaching of arithmetic in the schools, not many were written in the field.

In Colonial times texts used on this side of the Atlantic were almost entirely of English origin or, if written by American authors, were patterned after English texts. The first definite break away from the method of presentation used in these early arithmetics did not take place until more than 30 years after children were being taught the subject in the public schools of our country.

Through the 150-year period since arithmetic was made an official part of the curriculum of our schools, many changes have taken place in the textbooks in the field. Some of these have proved to be based on sound philosophy and have added much to the improvement

[1] Monroe, Paul. *A Cyclopedia of Education*, vol. I, p. 205. The Macmillan Co., New York, 1911.

[2] Newcomb, Ralph. *Modern Methods of Teaching Arithmetic*. Houghton Mifflin Co., Boston, 1926, p. 3.

of our texts. Others have apparently been made purely as an experiment and, having been found wanting, have been revised or dropped entirely. Some changes have come in waves, first swinging to one extreme and then to the other, with as yet no apparent agreement among educators as to what is best. And still others have shown a constant trend in the same direction—sometimes slow, sometimes fast.

During this time changes have taken place in almost every phase of the physical make-up, content material, and method of presentation of elementary arithmetics. While many factors have been instrumental in bringing about these changes, probably the two greatest influences have been the findings of child psychologists in studying the mental growth of children, their learning habits, interests, and abilities, and the desire on the part of textbook writers to produce arithmetics with both appeal and practicality. These two factors have of course gone hand in hand, with neither able to produce good results without the help of the other. Together, they have accomplished much.

Books Analyzed in This Study

The 59 representative arithmetic books or series of books that have been analyzed in previous studies by the Bureau of Cooperative Research and Field Service show many of these changes and have therefore been used as a basis for the present study.

In these analyses each of the series was given a number, with the texts arranged in order according to the date of publication. However, in some cases the date of original publication was used, while in others the date of the edition analyzed was used. This discrepancy resulted from the fact that the five studies were carried on over a period of years and were directed by different individuals.

In the present study the date of original publication is used throughout, although in a few cases this indicated that a text was published a number of years before the date given in the earlier Bureau analyses. While it is recognized that improvements in appearance and minor techniques were made in the later editions of a text, the basic principles concerning content and method were not greatly altered. Therefore the date of original publication seems to give a truer picture of how the text fitted in with the theories of educators of the day and what part it played in developing trends or adding new ideas in the teaching of arithmetic.

In the present analysis of the texts, the same number is used to designate each text as was used in the earlier studies. When a textbook is published in more than one volume, the most elementary of the books is listed as "a," the next most elementary as "b," and so on. The list of texts included in the study,[3] with their number and letter designations, follows:

[3] When material from any of these texts has been quoted or photographed in this study, the book from which it was taken has been indicated by the number of the text, followed by the page on which the material occurs. For example, a reference on page 121 of the fourth grade text in the *Child-Life Arithmetics* would be identified as (57b:121). In all cases in which material was taken from books under copyright, the permission of the publishers holding the copyright has been granted.

1. Cocker, Edward. *Cocker's Arithmetick.* Printed by Eben. Tracy, London, 1677. (1706 edition used)
2. Walkingame, Francis. *The Tutor's Assistant.* A. Millar, W. Law, and R. Cater, London, 1751. (1792 edition used)
3. Pike, Nicholas. *A New and Complete System of Arithmetic.* Printed at the press of Isaiah Thomas, Worcester, Mass., 1788. (1797 edition used)
4. Gough, John. *Practical Arithmetick.* Printed by W. Magee, Belfast, 1801.[4]
5. Pike, Nicholas. *The New Complete System of Arithmetick.* Published by Thomas and Andrews, Boston, 1797. (1807 edition used)
6. Pike, S. *The Teachers' Assistant.* Published by Johnson and Warner, Philadelphia, 1811.
7. Watson, Thomas; Jaudon, Daniel; and Addington, Stephen. *The Youth's Arithmetical Guide.* Printed by David Hogan, Philadelphia, 1805. (1814 edition used)
8. Daboll, Nathan. *Daboll's Schoolmaster's Assistant.* Printed by Thomas M. Skinner, Auburn, 1799. (1820 edition used)
9. Colburn, Warren. *Intellectual Arithmetic.* Oliver L. Sanborn, Concord, N.H., 1821. (1837 edition used)
10. Thompson, Zadock. *The Youth's Assistant in Practical Arithmetic.* David Watson, Woodstock, Vt., 1825.
11. Hassler, F. R. *Elements of Arithmetic.* G. & C. & H. Carvill, New York, 1826. (1832 edition used)
12. Smiley, Thomas T. *The New Federal Calculator, or Scholar's Assistant.* Lippincott, Grambo, and Co., Philadelphia, 1828. (1851 edition used)
13. Ostrander, Tobias. *The Elements of Numbers, or Easy Instructor.* Morse and Harvey, Canandaigua, N.Y., 1829. (1834 edition used)
14. Guthrie, Jesse. *The American Schoolmaster's Assistant.* L. Collins and B. Cox, Maysville, Ky., 1810. (1834 edition used)
15. DeMorgan, Augustus. *The Elements of Arithmetic.* John Taylor, London, 1832. (1835 edition used)
16. Talbott, John L. *The Western Practical Arithmetic.* E. Morgan and Co., Cincinnati, 1833. (1844 edition used)
17. Adams, Daniel. *Adams' New Arithmetic.* Collins and Brother, New York, 1827. (1841 edition used)
18. Chase, Pliny E. *The Common-School Arithmetic.* A. Hutchinson and Co., Worcester, Mass., 1848.
19. Perkins, George R. *An Elementary Arithmetic.* D. Appleton and Co., New York, 1849. (1866 edition used)
20. (Davies Series)
    - 20a. Davies, Charles. *First Lessons in Arithmetic.* A. S. Barnes and Co., New York, 1852. (1871 edition used)
    - 20b. Davies, Charles. *Arithmetic.* A. S. Barnes and Co., New York, 1849. (1852 edition used)
21. Dodd, James B. *Elementary and Practical Arithmetic.* Pratt, Oakley and Co., New York, 1849. (1854 edition used)
22. Eaton, James S. *The Common School Arithmetic.* Taggard and Thompson, Boston, 1863. (1866 edition used)
23. Colenso, The Right Rev. J.W., D.D. *Arithmetic.* Longmans, Green, and Co., London, 1864. (1868 edition used)
24. Felter, S. A. *The Analysis of Written Arithmetic.* Charles Scribner and Co., New York, 1862. (1864 edition used)

[4] When the edition used in this study is not indicated, the first edition was analyzed.

25. Dean, Philotus. *The Intermediate Arithmetic.* A. H. English and Co., Pittsburgh, 1865.
26. Greenleaf, Benjamin. *Introduction to the National Arithmetic.* Robert S. Davis and Co., Boston, 1842. (1870 edition used)
27. Peabody, Selim H. *Felter's New Practical Arithmetic.* Charles Scribner's Sons, New York, 1872.
28. Peck, William G. *Complete Arithmetic.* A. S. Barnes and Co., New York, 1874. (1875 edition used)
29. (French Series)
    - 29a. French, John H. *First Lessons in Numbers.* Harper and Brothers, New York, 1866. (1874 edition used)
    - 29b. French, John H. *Common School Arithmetic.* Harper and Brothers, New York, 1869. (1875 edition used)
30. Hagar, D. B. *A Common School Arithmetic.* Cowperthwait and Co., Philadelphia, 1871. (1877 edition used)
31. (Mecutchen-Sayre Series)
    - 31a. Mecutchen, S., and Sayre, G. M. *The New American Arithmetic.* E. H. Butler and Co., Philadelphia, 1877.
    - 31b. Mecutchen, S., and Sayre, G. M. *The New American Practical Arithmetic.* J. H. Butler and Co., Philadelphia, 1877.
32. Milne, William J. *The Practical Arithmetic.* Jones Brothers and Co., Chicago, 1877. (1879 edition used)
33. Raub, Albert N. *The Complete Arithmetic.* Porter and Coates, Philadelphia, 1877.
34. (Ray Series)
    - 34a. Ray, Joseph. *Ray's Elementary Arithmetic.* Van Antwerp, Bragg and Co., New York, 1879.
    - 34b. Ray, Joseph. *Ray's New Practical Arithmetic.* Van Antwerp, Bragg and Co., New York, 1877.
35. Baker, Andrew H. *Elementary Arithmetic.* P. O'Shea, New York, 1878.
36. Olney, Edward. *A Practical Arithmetic.* Sheldon and Co., New York, 1879.
37. (White Series)
    - 37a. White, E. E. *A New Elementary Arithmetic.* Van Antwerp, Bragg, and Co., New York, 1870. (1883 edition used)
    - 37b. White, E. E. *A New Complete Arithmetic.* Van Antwerp, Bragg, and Co., New York, 1870. (1883 edition used)
38. (Indiana Educational Series)
    - 38a. Blythe, James E. *Elementary Arithmetic.* Indiana Schoolbook Co., Indianapolis, 1886. (1889 edition used)
    - 38b. Blythe, James E. *Complete Arithmetic.* Indiana Schoolbook Co., Indianapolis, 1886. (1889 edition used)
39. (Milne Series)
    - 39a. Milne, William J. *Elements of Arithmetic.* American Book Co., New York, 1893.
    - 39b. Milne, William J. *Standard Arithmetic.* American Book Co., New York, 1892.
40. (Southworth Series)
    Southworth, Gordon Augustus. *The Essentials of Arithmetic.* Thos. R. Shewell and Co., New York, 1893-95.
    - 40a. *Book I, for Lower Grades,* 1893.
    - 40b. *Book II, for Upper Grades,* 1895.
41. (Walsh Series)
    Walsh, John H. *Mathematics for Common Schools.* D. C. Heath and Co., Boston, 1895.
    - 41a. *Part I—A Primary Arithmetic.* (1898 edition used)
    - 41b. *Part II—Grammar-School Arithmetic.* (1897 edition used)

42. (McLellan-Ames Series)
    42a. McLellan, J. A., and Ames, A. F. *The Primary Public School Arithmetic.* The Macmillan Co., New York, 1898.
    42b. McLellan, J. A., and Ames, A. F. *The Public School Arithmetic.* The Macmillan Co., New York, 1898.

43. (Indiana Schoolbook Series)
    43a. Hornbrook, A. R. *Indiana Elementary Arithmetic.* Indiana Schoolbook Co., Indianapolis, 1898. (1899 edition used)
    43b. Bailey, M. A. *Indiana Advanced Arithmetic.* Indiana Schoolbook Co., Indianapolis, 1897. (1899 edition used)

44. (Woodward Series)
    44a. Luckey, E. D. *Elementary Arithmetic.* Woodward and Tiernan Printing Co., St. Louis, 1899.
    44b. Luckey, E. D. *New Practical Arithmetic.* Woodward and Tiernan Printing Co., St. Louis, 1899.

45. (Hornbrook Series)
    45a. Hornbrook, Adelia R. *A Primary Arithmetic.* American Book Co., New York, 1898.
    45b. Hornbrook, Adelia R. *Grammar School Arithmetic.* American Book Co., New York, 1900.

46. (Johnson Series)
    Colaw, John M., and Ellwood, J. K. *School Arithmetic.* B. F. Johnson Publishing Co., Richmond, Va., 1900.
    46a. *Primary Book.*
    46b. *Advanced Book.*

47. (Southworth-Stone Series)
    Southworth, Gordon A., and Stone, John C. *The Southworth-Stone Arithmetic.* Benjamin H. Sanborn and Co., New York, 1904.
    47a. *Book I, for Lower Grades.*
    47b. *Book II, for Higher Grades.*

48. (Smith Series)
    48a. Smith, David Eugene. *Primary Arithmetic.* Ginn and Co., Boston, 1904.
    48b. Smith, David Eugene. *Grammar School Arithmetic.* Ginn and Co., Boston, 1904.

49. (Cook-Cropsey Series)
    49a. Cook, John Williston, and Cropsey, Miss N. *The New Elementary Arithmetic.* Silver, Burdett, and Co., New York, 1893. (1906 edition used)
    49b. Cook, John Williston, and Cropsey, Miss N. *The New Advanced Arithmetic.* Silver, Burdett, and Co., New York, 1892. (1906 edition used)

50. (Modern Arithmetic Series)
    Cox, Henry C.; Tallant, George A.; and Umholtz, Frederick H. *Modern Arithmetic.* Lyons and Carnahan, Chicago, 1913.
    50a. *Elementary* (by Cox).
    50b. *Advanced.*

51. (Everyday Arithmetic Series)
    Hoyt, Franklin S., and Peet, Harriet E. *Everyday Arithmetic.* Houghton Mifflin Co., Boston, 1915.
    51a. *Book One.*
    51b. *Book Two.*
    51c. *Book Three.*

52. (School Arithmetic Series)
    Wentworth, George, and Smith, David Eugene. *School Arithmetics.* Ginn and Co., Boston, 1919. (1920 edition used)
    52a. *Book One.*
    52b. *Book Two.*
    52c. *Book Three.*

53. (Progressive Arithmetic Series)
Smith, David Eugene. *Progressive Arithmetics.* Ginn and Co., Boston, 1924.
53a. *Primary Book.*
53b. *Intermediate Book.*
53c. *Advanced Book.*

54. (Modern Life Arithmetics Series)
Fowlkes, John Guy, and Goff, Thomas Theodore. *The Modern Life Arithmetics.* The Macmillan Co., New York, 1928. (1929 edition used)
54a. *Book One.*
54b. *Book Two.*
54c. *Book Three.*

55. (Triangle Arithmetics Series)
Brueckner, Leo J.; Anderson, C. J.; Banting, G. O.; and Merton, Elda L. *The Triangle Arithmetics.* The John C. Winston Co., Philadelphia, 1928.
55a. *Book One.*
55b. *Book Two.*
55c. *Book Three.*

56. (Iroquois Arithmetics Series)
DeGroat, Harry DeW.; Firman, Sidney G.; and Smith, William A. *The Iroquois Arithmetics.* Iroquois Publishing Co., Inc., Syracuse, New York, 1932.
56a. *Book One.*
56b. *Book Two.*
56c. *Book Three.*

57. (Child-Life Arithmetics Series)
Woody, Clifford; Breed, Frederick S.; and Overman, James R. *Child-Life Arithmetics.* Lyons and Carnahan, Chicago, 1936-37.
57a. *Grade Three,* 1937.
57b. *Grade Four,* 1937.
57c. *Grade Five,* 1937.
57d. *Grade Six,* 1937.
57e. *Grade Seven,* 1936-37.
57f. *Grade Eight,* 1936-37.

58. (Daily-Life Arithmetics Series)
Buswell, Guy T.; Brownell, William A.; and John, Lenore. *Daily-Life Arithmetics.* Ginn and Co., Boston, 1938.
58a. *Grade Three.*
58b. *Grade Four.*
58c. *Grade Five.*
58d. *Grade Six.*
58e. *Grade Seven.*
58f. *Grade Eight.*

59. (Study Arithmetics Series)
Knight, F. B.; Ruch, G. M.; Studebaker, J. W.; and Findley, W. C. *Study Arithmetics.* Scott, Foresman and Co., Chicago, 1934-37.
59a. *Grade Three,* 1934. (1940 edition used)
59b. *Grade Four,* 1935. (1940 edition used)
59c. *Grade Five* (by Knight, Studebaker, and Ruch), 1935. (1940 edition used)
59d. *Grade Six* (by Knight, Studebaker, and Ruch), 1936. (1940 edition used)
59e. *Grade Seven* (by Knight, Studebaker, and Ruch), 1937. (1942 edition used)
59f. *Grade Eight* (by Knight, Studebaker, and Ruch), 1937. (1942 edition used)

The only criterion that could be used in selecting texts representative of the first half century covered in this study was that of availability, for not many of the arithmetics published in those early days are in existence today. It is believed, however, that the books analyzed here are typical. Later texts included were selected according to what was popular at the time of their publication. Without question, many other good arithmetics were written, but analysis had to be based on a representative few. Any generalizations or trends in textbook style or procedure pointed out in this study are based solely on the texts analyzed, but are thought to be reasonably true of all arithmetics of the times.

Among the texts used are arithmetics of every period from 1790 to 1940. The oldest book in the group, *Cocker's Arithmetick*, was first published in 1677, more than a century before the beginning of the period being studied, but it has been included here as an example of the arithmetics of English origin from which our texts developed. Another text, Walkingame's *The Tutor's Assistant* (1751), was also published long before 1790. This book is included because it, also, was among those analyzed in the earlier Bureau studies, the 1792 edition having been used. The most recent arithmetics analyzed were published or revised within the last year or two covered by the study. While many phases of the content of these 59 books have already been presented in tabular form in the Bureau studies,[5] the discussion has been confined to a specific 30-year period instead of to one continuous period covering the entire 150 years. For this reason some of the more interesting trends have not been shown clearly.

[5] These analyses have been published under the title of *Analysis of Arithmetic Textbooks* in issues of the Bulletin of the School of Education, Indiana University. These issues are as follows: *First Period—1790-1820*, vol. XVIII, no. 1, January, 1942; *Second Period—1821-1850*, and *Third Period—1851-1880*, vol. XVIII, no. 6, November, 1942; *Fourth Period—1881-1910*, vol. XIX, no. 4, July, 1943; and *Fifth Period—1911-1940*, vol. XIX, no. 6, November, 1943.

Chapter II
Purposes

In the arithmetics analyzed in this study, the underlying purpose—to teach arithmetic that is practical—has remained unchanged through the years. Many factors have entered in, however, to alter the interpretation of practicality in arithmetic, and, as the point of view has changed, the specific aims and objectives mentioned by the authors have varied somewhat. Since authors usually stated in their prefaces in what ways they believed their texts met the needs of the day, probably the most accurate idea of the larger movements in purpose may be gained through a study of prefaces in the different texts.

Authors of most of the earliest texts were consciously catering to those who needed instruction in the mathematics of their business dealings. Cocker (1677) made this especially clear in the preface to his *Arithmetick* when he stated, "I do ingeniously profess, that in the beginning of this Undertaking, the numerous Concerns of the honoured Merchants first possessed my Consideration," because, as he said, "as Merchandise is the Life of the Weal-Publick, so Practical Arithmetick is the Soul of Merchandize" (1:A6). Cocker also stated that he had prepared his text for professors, students, and those interested in number as a science, and he presented it to each group individually. In presenting it to the teachers, for instance, he said:

> For your Service, most excellent Professors, whose Understandings soar to the Sublimity of the Theory and Practice of this Noble Science, was this Arithmetical Tractate composed; which you may please to imploy as a Monitor to instruct your young TYROES, and thereby take occasion to reserve your precious Moments, which might be exhausted that way, for your more important affairs. (1:A6)

Since arithmetic was taught primarily for the benefit of business men, it is understandable that the vocabulary used was very technical, and the sentence structure, from a modern point of view at least, was often rather awkward and involved. Apparently no attempt was made to meet public demand so far as writing a text for the average man was concerned.

One of the problems which concerned authors of these early texts was the time-consuming task of the teacher in "setting sums" for the pupils to copy. When pupils did not have texts of their own, the teacher wrote out problems and their solutions, which the students were to copy in their "sum books." The teacher's task was not only to prepare the material, but also to judge the neatness and correctness of the pupils' work. With this problem in mind, many of the authors seemingly wrote their texts solely for the purpose of saving the teacher's time. Walkingame (1751), who was among these, went into detail in his explanation of how his text would save the time of the teacher, because "when a master takes upon him that laborious (though unnecessary) method of writing out the rules and questions in the children's

books, he must either be toiling and slaving himself, after the fatigue of the school is over, to get ready the books for the next day, or else must lose that time which would be much better spent in instructing and opening the minds of his pupils . . . ," and of how it would save the time of the pupils, because "where there are several boys in a class, some one or other must wait till the boy who first has the book finishes the writing out those rules or questions he wants, which detains the others from making that progress they otherwise might, had they a proper book of rules and examples for each; . . ." (2:iii-iv).

Now and then a teacher would write a text for his own use, thus solving the problem of finding an arithmetic that would suit his needs. While he would design the book for his own students, he would, of course, make it available to others who might wish to use it. *The Youth's Arithmetical Guide*, written in 1805, was a typical example of this kind of arithmetic. The authors dedicated it thus:

TO
THE YOUTH OF PHILADELPHIA,
Particularly to those
UNDER THE CARE OF THE AUTHORS,
The Following System
of
PRACTICAL ARITHMETIC,
Is respectfully dedicated.

Young Ladies and Gentlemen,

WE, who have the pleasure of addressing you on the present occasion, have, as many of you well know, devoted much time and labour to the instruction of youth; and we present you with the following compend, as an additional evidence of our good wishes, and our earnest solicitude for your interest. (7:iv)

It was not uncommon in early days for boys entering some kind of apprenticeship or business to study at home, either alone or in groups, with only the aid of a textbook. Texts were sometimes designed especially to meet the needs of these individuals, although they could also be used in the classroom. In announcing that his book was valuable for use without the aid of a teacher, Hassler, in his preface, expressed the desire to see a group of boys "passing an evening together, with this book among them, each his slate and pencil before him, discussing, mutually giving and solving, the questions which they learn from it, to form out of the occurrences around them. I can promise them more satisfaction from it than passing that time in the barroom of a public house" (11:ix). He went on to discuss the point at great length. (This would seem a rather extreme change of interests to hope for on the part of those boys, especially since the book was dull in appearance both inside and out.)

Even as late as 1866 French stated that his text was designed for use "in classes or families," but this was long after other authors had apparently given up all idea of writing a text for use at home. Since French's arithmetic was an extremely elementary one (the first in a series of four), it may be that he was considering the families

who lived too far from school to send the small children alone, and who therefore taught them at home.

During these early years arithmetics had been used as reference books and were filled with rules and definitions that would provide information aiding in the solution of almost any problem that might arise in one's business. Smiley (1828) made it clear that his purpose was to provide such a text when he gave it the subtitle:

> Containing the Most Concise and Accurate Rules for Performing the Operations in Common Arithmetic; Together with Numerous Examples Under Each of the Rules, Varied so as to Make Them Conformable to Almost Every Kind of Business; for the Use of Schools and Counting Houses. (12:title page)

One of the last authors to stress practicality from the point of view of the businessman was Peck, who said his aim was "to present, within moderate limits, all the principles of arithmetic required by the student, the man of business, and the artisan" (28:3).

After 1821 a distinct effort was made to write textbooks in a simpler form, so that younger children could understand them. This was an outgrowth of a movement started by Colburn to provide texts for young children on the inductive instead of the deductive plan. Many authors believed firmly in the advantages of such a presentation and, in writing textbooks, set out as one of their purposes to provide an arithmetic that was understandable to the young child, a practice that has been followed up to the present time. Although the attempt to simplify texts had become very much in evidence by 1875, it had created considerable criticism from some quarters, as was evidenced by statements in some of the texts. Mecutchen and Sayre (1877), for instance, called their text "a working text-book, and not a pictured play-book" (31a:2), and Olney (1879) stated that his was a "book of work" (36:4) to meet the practical wants of the schoolroom.

Watson *et al.* were the first to mention that "every endeavor has been made to have the whole as level as possible to the capacities of those for whom it is designed" (7:vi). A little later, after Colburn published a text specifically for the young child, this became a feature of most texts. In recent years an effort has been made to keep the work for each grade on a level scientifically determined as within the ability of pupils of that age.

Throughout the early years covered by this analysis, there was a tendency for authors to state their purposes in rather general terms, but by the middle of the century there was evidence of an attempt to be more specific in this matter. Davies (1852), for example, said that his elementary arithmetic was designed to teach children to count, to combine numbers, and to read numbers. The use of greater detail here probably came as a result of dividing texts into two or more volumes, for in this way the aims and objectives on the various grade levels were necessarily somewhat different and therefore had to be stated more specifically. In recent texts these vary considerably from grade to grade.

Whenever certain teaching theories were particularly popular, authors often stated in their prefaces that they had designed their

texts in such a way as to provide for putting these theories into practice. For example, when the theory of mental discipline was generally accepted, a number of authors claimed to have presented their material in such a way as to develop the minds of the pupils. Greenleaf (1842) was among the first to state this purpose clearly: "the object of studying mathematics is not only to acquire a knowledge of the subject, but also to secure mental discipline, to induce a habit of close and patient thought, and of persevering and thorough investigation" (26:iv). He then explained how the use of his book would aid in the accomplishment of this result. Similar comments were found in prefaces of texts published for 40 or 50 years after this, though they were not always given in as great detail. Later, the mental discipline theory lost favor, and comment about the value of a text from this point of view was dropped from arithmetics.

Greenleaf was the first, and one of the very few, authors to state that one of the purposes of his text was to inspire in the child "a love for mathematical science" (26:iii). Many, however, followed Greenleaf's lead in attempting to provide an arithmetic that would interest the pupil. Perkins (1849), for one, stated that:

> We are confident that the present work will maintain a longer than usual hold on the interest of both teachers and pupils; for it is not, like a cistern, to be exhausted by a few drawings, but like nature's reservoirs, it has the fountain within itself. (19:iii)

Perkins' text was small and rather uninteresting in appearance, with a great deal of printed matter on a dull gray cover, but the inside pages of the book were a decided improvement over those of older arithmetics. The content was not crowded, the type was larger than usual, and there were fewer kinds of type used. No pictures, except for a few line drawings used for instructional purposes, were included.

Much progress has been made since that time in developing texts that would interest children. Textbook writers have been well aware of the need for securing this interest, and have tried various ways in which to gain it, such as including attractive pictures in color, explaining new material in storybook fashion, using "catchy" headings, etc.

Up to about 1880 most texts had contained a "complete" course. No provision had been made for those who dropped out of school early and consequently they often left without having received some of the elementary basic principles of arithmetic. Some authors tried to meet this situation by rearranging topics and placing the most essential ones early in the text. Raub (1877), however, was the first of these authors to suggest a textbook to meet the need for a shorter course. The attempt to give children a foundation in arithmetic, so that they would be prepared for their work in life if they had to leave school early, was found stated as an aim in texts even within the last 30-year period in this analysis.

Interest in developing skill in the fundamentals came with interest in mental discipline, and remained as an important purpose of texts until long after the turn of the century. The Milne series (1892-93)

stated as one of its objectives to help the student to attain "skill in numerical computations, and a proper understanding of the reasons for the steps in the explanation of processes and the solution of problems" (39b:3).

As the popularity of the theory of mental discipline waned, in the 1890's, authors often mentioned their desire to increase understanding of principles and processes. Thus skill in the fundamentals and thorough understanding of the reasoning involved in arithmetic shared honors as important purposes of texts for a number of years. The Southworth series (1893-95) stated that it aimed to train "the pupil in thinking and reasoning about matters involving the use of numbers" (40a:iii), and the Hornbrook series (1898) expressed this especially clearly in the prefaces to both its elementary and its advanced books —to secure a "succession of insights into the relations of numbers and an increase of expertness in dealing with them" (45a:3), and "to secure a ready skill in dealing with numbers and to develop thought power adequate to the attack of any arithmetical problem that may arise in practical life" (45b:4). These purposes were still predominant in 1913 when the *Modern Arithmetics* were published.

Although as early as 1899 there had been statements in prefaces to the effect that an attempt was made to help pupils apply the fundamental processes in arithmetic to the ordinary affairs of life, this was not found stated as a purpose until 1915, when the *Everyday Arithmetics* gave as one of their objectives "to give a child an active interest in number and to make him intelligent and skillful in applying it to his everyday experiences" (51a:iii). Here also was one of the first real attempts to put arithmetic into life experiences.

The *Iroquois Arithmetics* (1932) were the first of those analyzed to attempt "to create in the pupil's mind a desire to use numbers and to supply the means of satisfying that desire" (56c:iii), and later the *Child-Life Arithmetics* (1936-37) gave as one of their aims that of creating a need in the minds of the pupils, thus developing in the children a desire to find out how to solve the problems.

The social aspects of arithmetic began to be emphasized about this time, and most of the recent authors stated that they had attempted to adapt their arithmetics to the social demands made on the child after he leaves school. The *Child-Life Arithmetics* brought out as a purpose that of presenting new material in social situations familiar to children, in order to make the need for arithmetic in out-of-school life felt.

The *Daily-Life Arithmetics* (1938) gave as their purpose "to develop intelligence in dealing with number,—in the child now, in the adult later" (58b:v), and to "make children (and adults) sensitive to the quantitative aspects of our culture" (58b:v). In the *Study Arithmetics* (1934-37) the purpose was given more specifically: "(1) to develop essential third-grade number facts and concepts in a carefully organized series of interesting stories and exercises; (2) to give specific training in interpreting the language of arithmetic; and (3) to build a reading familiarity with the necessary terminology, symbols, and forms" (59a:3). This text also stated that "the opportunity is created

for social enrichment, the development of mathematical insight, and the appreciation of mathematics as a reality in the living world" (59b:3).

And so today, just as in the early years covered by this study, the fundamental purpose of authors in writing arithmetic textbooks is that of giving the student the information he can use now or will probably need when he leaves school. At first, the present and future needs were very similar, for the student was usually an adult, and, as a rule, the arithmetic taught was in the field of business because the student ordinarily studied the subject as preparation for handling his work. Nowadays the student of elementary arithmetic is almost always a child, and therefore the arithmetic that he needs is quite different from that taught earlier. However, the purposes of authors are still based on the idea of giving students practical arithmetic, the kind they can use in their daily experiences, and to present it in such a way that their needs will have been met by the time they have completed the course. In fact, recent authors could well state, in modern terminology, of course, the same thought in their prefaces today as was expressed by Cocker, more than two and a half centuries ago:

> This Arithmetick ingeniously observed, and diligently practised, will turn to good account to all that shall be concerned in Accompts. All whose Rules are grounded on Verity and delivered with Sincerity. The Examples are built up gradually from the smallest Consideration to the greatest. All the Problems or Propositions are well weighed, pertinent, and clear, and not one of them throughout the Tract taken upon trust, therefore now,
>
> Zoilus and Momus lie you down and die,
> For these Inventions your whole force defie. (1:A7)

to appear on the outside. Between 1828 and 1890 only four of the 27 books studied were bound in anything other than paper-covered cardboard. The earliest of these four was covered in black leather (1832) and the others in cloth.

The idea of cloth-covered cardboard for book covers gained in popularity almost as rapidly as did its predecessor, its greater durability being a decided advantage over the paper cover. While only three of the books before 1890 had cloth covers, during the last 50 years every one of the arithmetics studied was bound in this way. Among these cloth-covered books, Olney's *A Practical Arithmetic*, published in 1879, with a backbone of leather, was the only one in which the back binding was not of the same material as the cover.

The leather covers of the early books were usually tan or brown, occasionally black. As a consequence, even when other materials came into use, shades of brown or gray continued to be popular. The first colors to be used on the paper covers were tan, beige, gray, dusty rose, and pale gray-blue. Seldom was a deep color used but, if used, a dark, dulled shade of the color was chosen rather than one which would make the textbook look gay. In 1898 the Macmillan Company published the first book of the McLellan-Ames series in deep rose red and the second book in dark olive green, and in the same year the American Book Company put out the Hornbrook series in a rich dark green. These books helped to pave the way for a greater use of color, but it was not until 1928, when the John C. Winston Company published the *Triangle Arithmetics* in a clear light blue, that any of the textbook covers were free from a toned-down, grayed effect. Four years later, the Iroquois Publishing Company, Inc. ventured forth with a set of arithmetics bound in an "exciting" bright red. In 1938 Ginn and Company broke even further away from tradition and published a set of six arithmetics, each in a different color. These books, designed for the third to the eighth grade, were bound in orange, red tan, dark blue green, light green, bright red, and bright blue, respectively.

Publishers were even slower in using colored ink to brighten the covers of arithmetics than they were in using color in material. Nothing but black ink was used until about 1900, when the Johnson series was printed in dark brown, and, from that time until 1940, black alone was used in printing the covers of only two of the books studied. In most cases where color was used, a darker shade of the color of the cover was selected, but on some of the more recent books the color of the ink was in contrast to that of the cover, and on a few it was in different colors on different volumes of the series.

Because of the material used for the early covers, there was no printing on the outside of the books. The copy of *Cocker's Arithmetick* (1677) used in this study had evidently been rebound in the early days of paper-covered cardboard covers; its backbone was leather, but the front and back covers were in an all-over wavy two-tone effect in brown and gray—a modest, serious-looking little book. *The Youth's Arithmetical Guide* (1805), with a lacy gold border imprinted on a saddle brown leather, had the most elaborate of the early covers.

The first print on the covers of any of these texts was used by

Colburn in 1821, and soon after this all covers were printed. Many authors went to the extreme of using the opportunity not only to give the main title, subtitle, and other information about the arithmetic text, the name and position of the author, the publisher and place of publication, and so forth, but also to advertise other books by the same author or publisher. When this fad was at its height, the covers of some of the books were printed almost as solidly and in as small print as the pages within the book.

The solidly printed covers of the middle 1800's appeared heavy and must have seemed forbidding to the young mind. Textbook writers began to realize this, and gradually the amount of information given on the outside of a book was reduced to only the main title, author, and publisher. Some of the covers today give even less information than this, as can be seen in Figure 2.

Pictures were first used on covers about 1860. Many of the first books to venture to illustrate the cover used only a small head or crest, accompanied by printed matter. Where larger pictures were used, they often concerned children, but always in a schoolroom situation. Not until after 1910 did a textbook writer become daring enough to picture children playing, but this did not become a common practice until the late 1930's.

Shortly after the middle of the nineteenth century, borders were sometimes used on covers, and somewhat later, in the first part of the twentieth century, conventional designs were very popular. As the trend toward simplification of the covers became apparent, authors often illustrated only the top half of the page, leaving the bottom half blank or including on it only the name of the book or the volume number. Today the page has become more balanced, with active children pictured enjoying themselves at work or play.

On the whole, both the early and the late arithmetics had plain back covers, but the publishers of texts between 1820 and 1880, when printed covers were most popular, sometimes took advantage of this space to advertise their other publications. They listed textbooks in other subjects put out by the same publisher, or other texts by the same author, informing teachers and other interested parties of the available material in the field. In some cases recommendations were printed on the backs of the books. Smiley and Ostrander, whose books appeared in 1828 and 1829, respectively, were outstanding examples in this respect. These recommendations will be discussed more fully later.

After about 1895, most of the arithmetics had rather plain backs, giving, if anything, only the seal or the name and address of the publisher and the price of the book. The only books with an illustration printed on the back were French's *First Lessons in Numbers* (1866), *The School Arithmetics* (1919), and the *Daily-Life Arithmetics* (1938). As can be seen, the first of these texts was more than 50 years ahead of its time in this respect. The illustrations were those of several groups of articles—five leaves, seven bottles, ten cherries, etc.—with the figures and words describing each written in three different ways around the picture for the purpose of teaching the pupil how to recognize mathematical concepts in print. The second of the books with

illustrations on the back had a small square of nine numbers in the center, and the last one used the rather novel idea of completing the picture on the front. For example, the volume picturing a boy riding a pony in cowboy fashion and lassoing arithmetical symbols, shown in Figure 2, pictures, on the back cover, the same boy, his work accomplished, waving his hat in farewell, and riding away with the four symbols strapped to the back of his saddle.

The inside covers of by far the largest percentage of the books were plain white. During the time when printed matter was so popular on the outside of the cover, six books carried this idea into the inside also, printing advertisements on these pages. Greenleaf's book had the title page printed on the inside of the front cover and Mecutchen-Sayre's elementary text had tables and measures on the inside of the back, but no other book used the inside of the covers for any purpose whatsoever until the *Iroquois Arithmetics*, in 1932, covered these pages with pictures of little goblins representing various difficult number combinations. Two of the three series included in this study after that date also illustrated these pages.

Number of Volumes in Texts

As has been mentioned earlier, before arithmetic had become a well-organized subject in the school curriculum, textbooks were written for those needing the knowledge for business purposes or for those who enjoyed the study of arithmetic as a science. Consequently, there was no cause for grading the texts other than to give the fundamentals necessary to each new step presented. Very little elementary work was included in these books, and it was not until 1849 that an author of any of the texts studied was convinced that the more elementary work for beginners should be published in a separate volume from the more advanced work for older pupils.

In 1805 Watson *et al.* felt out the attitude of teachers on this point by publishing their text in two parts, each with a separate table of contents and with the two parts separated by two pages of advertising matter. However, they were not quite sure enough of themselves to call the matter to anyone's attention, and they numbered the pages consecutively from beginning to end, thus concealing their venture to a certain extent.

Apparently no one else took up with the idea until DeMorgan, in 1832, included two books under one cover, the second a text in commercial arithmetic. No break was made in the volume, except that the second book began on a new page and the numbered sections within each book began with "I." Article numbers and page numbers were consecutive from the beginning of the first part to the end of the book.

In 1849 Davies published the first two-book series of arithmetics included in this study. It was 25 years before any other author, except Perkins,[6] who put one out the same year, attempted to grade their

[6] Only the elementary text is included in this study.

texts in this way. Then Peck published a two-book series and French went still further, writing a four-book series.[7]

Three other two-book series appeared between that time and 1880, after which date none of the authors of the arithmetics studied attempted to present their material in one volume only. A number of these started their elementary texts with work of the first grade, but the majority began with the third grade, stating that in their elementary texts they included a review of the arithmetic facts children in grades 1 and 2 should know. The first volume of these arithmetics usually included the fourth grade work, and a few took in the fifth grade also; the second volume covered the work of the remaining three or four grades.

In 1893-95 Benjamin H. Sanborn and Company published the *Southworth-Stone Arithmetics*, a series of two books that was really a set of three books, each designed for two grades. Parts I, II, and III, designed for grades 3, 4, and 5, were in the first book, and the other three parts in the second. However, the numbering of pages began anew with Part III, the first 118 pages being found in the first book and pages 119 to 236 (Part IV) being included in the first part of the advanced text. Part V again began with page 1, although it was in the middle of a volume, and the pages were numbered consecutively to the end of Part VI.

In 1915 the Houghton Mifflin Company published an arithmetic series in three volumes, covering two grades each. After the *Everyday Arithmetics* appeared, no others were found in fewer than three volumes, a fact which indicates that the grading of arithmetics was very practical from the point of view of both teachers and pupils. By 1934 the idea of separate texts for separate grades had expanded still further, and the last three arithmetic series included in this study were put out in six volumes, one for each grade from the third to the eighth. As a forerunner of the six-book series, the *Triangle Arithmetics* appeared in 1928 in three separate volumes, each containing two parts, paged separately. The policy for numbering pages in books containing separate volumes under the same cover was fairly consistent in the more recent books. However, the plan used was unsatisfactory in that, since the numbering began anew with the beginning of the work for each new grade, any reference to a certain page had to be identified according to the part of the volume in which it was found.

In almost all cases in which the text was published in more than one volume, the books within a series were the same color and approximately the same size. When the size of the texts varied, the smaller size was used consistently for the younger pupils. The more notable cases of variety in color have already been mentioned. Scott, Foresman and Company tried a new idea in varying the appearance of the different books in the *Study Arithmetics* (1934-37) by using a dark brown for

[7] Not all of these books were available. For this reason, the second book in Peck's series and the first and last in French's series are the only books in these series to be included in this study.

the covers of all the books but printing each in a different color. In design, the covers within a series were usually alike. There were no exceptions to this between 1886 and 1910, although there were a few differences both before and after this period. Even when books were not alike, a general similarity in style and arrangement was evident. The books in a series were, on the whole, of approximately the same length; if differences were noticeable it was always in the direction of larger books for the more advanced grades.

Titles of Texts

On the whole the main titles of the books were comparatively short, the earliest having the simple title of *Cocker's Arithmetick*, and the last series listed, the title of *Study Arithmetics*. Although brief, these titles were often indicative of the type of texts being written at the time—*The Tutor's Assistant* (1751), a text for the schoolmaster; *The Youth's Arithmetical Guide* (1805), for the pupil trying to teach himself; and *The Common-School Arithmetic* (1848), for children attending school. From 1850 until well past the end of the century titles were similar to this last one, by Chase, except that most of them also stressed the fact that they were "practical" or "complete" or "elementary."

In the first few two-book series each book had a separate main title, for example, *Ray's Elementary Arithmetic* and *Ray's New Practical Arithmetic*. However, the custom later developed of using one name for the series, and identifying each book within the series as "Primary Book" or "Advanced Book," "Book 1" or "Book 2," up to Book 6 in the recent texts. The first series to accept this plan was the Southworth series, called *The Essentials of Arithmetic* and published in 1893-95.

All through the period covered by this study, and especially in its early days, textbook writers felt called upon to qualify their book titles more in detail. For this purpose they had added subtitles containing information about the content, method, and group for whom the book was designed. By far the longest and most comprehensive subtitle appearing on the title page of any of the texts was that found in Walkingame's (1751):

THE
TUTOR'S ASSISTANT;
Being a
COMPENDIUM OF ARITHMETIC,
and
A COMPLETE QUESTION-BOOK.
Containing,

I. ARITHMETIC in WHOLE NUMBERS; being a brief explanation of all its Rules, in a New and more concise Method than any hitherto published; with an Application to each Rule, consisting of a large Variety of Questions in real Business, with their Answers annexed.

II. VULGAR FRACTIONS, which are treated with a great deal of Plainness and Perspicuity.

III. DECIMALS, with the EXTRACTION of the SQUARE, CUBE, and BIQUADRATE ROOTS, after a very plain and familiar Manner; in which are set down Rules for the easy Calculation of INTEREST, ANNUITIES and PENSIONS IN ARREARS, the PRESENT WORTH of ANNUITIES, &c. either by Simple or Compound Interest.

IV. DUODECIMALS, or MULTIPLICATION of Feet and Inches, with Examples applied to measuring and working by Multiplication, Practice, and Decimals.

V. A COLLECTION OF QUESTIONS set down promiscuously, for the greater Trial of the foregoing Rules.

TO WHICH ARE ADDED,

A new and very short Method of extracting the CUBE-ROOT, *and a* GENERAL TABLE *for the ready calculating the* INTEREST *of any Sum of Money, at any Rate per Cent. likewise Rents, Salaries,* &c.

The whole being adapted either as a QUESTION-BOOK for the Use of Schools, or as a REMEMBRANCER and INSTRUCTOR to such as have some Knowledge therein. (2:title page)

Later, when the understanding of rules rather than mere memorization was being stressed, and when practical arithmetic for the children of the country was urged, subtitles similar to the following were found.

Wherein the rules are illustrated, and their principles explained: Containing a great variety of exercises, particularly adapted to the currency of the United States: to which is added a short system of bookkeeping, designed for the use of schools and private students. (16:title page)

By the middle 1800's a tendency to make subtitles much shorter and more general was evident, and such statements were reduced to "Combining the elements of the science, with their practical applications to business" (29b:title page) or "Combining oral and written exercises in a natural and logical system of instruction" (33:title page). This trend continued until titles were used only to indicate whether the book was for the elementary or upper grades and, finally, to designate the exact grade for which it was written. Since such a statement is very brief and uninformative, it has become more of an identification than a subtitle. It could probably be said with reason, therefore, that few texts have had subtitles in the true sense of the

word since early in the present century. Most of the information formerly given in subtitles is now found in the preface.

No apparent originality was noted in naming books, nor was there any evidence of an attempt to make book titles sound interesting until the last 30 years of the study. Then such descriptive words as "everyday" and "modern life," still stressing the practical side of arithmetic, came into use. The *Child-Life Arithmetics*, published in 1936-37, seemed more successful than any of the others in their attempt to get away from time-worn titles and yet continue the custom of indicating through the title that the book was both elementary and practical.

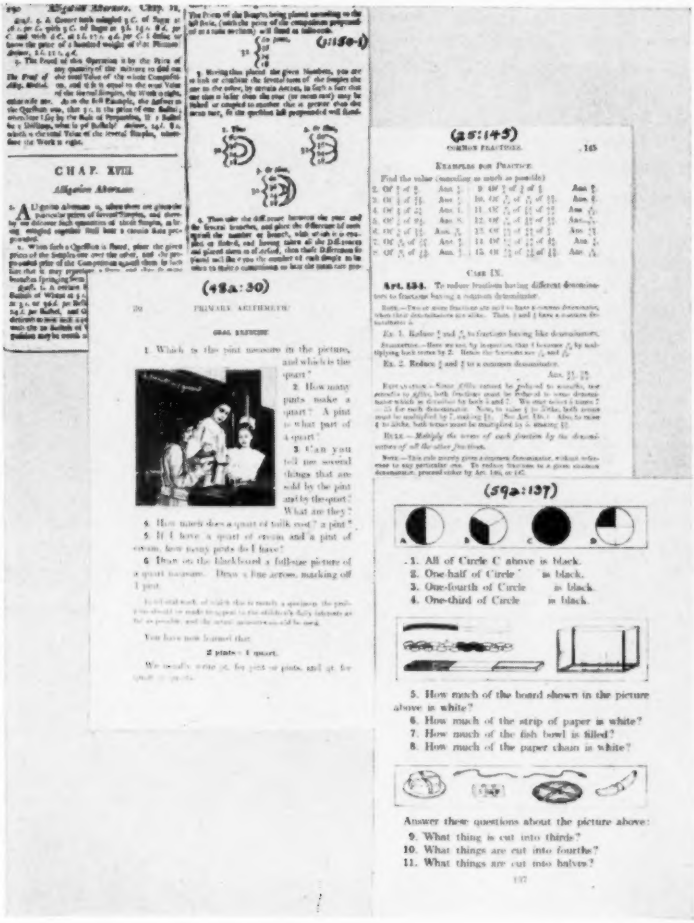
Appearance of Page

No trends were apparent in the number of pages in the arithmetic books. Most of the early books were shorter than the more recent ones, and, of course, all were shorter if the total number of pages in a series was considered. On the whole, each separate book came within the range of 200 to 400 pages, more recently published books being slightly longer than early ones. Great discrepancies occurred, however—for example, one of the first texts used, Pike's *A New and Complete System of Arithmetic* (1788), was one of the largest, with 515 pages, while Mecutchen and Sayre's beginner's book (1877) had only 84.

There was far more variation in the appearance of the page than in the number of pages in these arithmetics. The work of the child psychologist has probably made a greater difference here than in any other phase of the physical make-up of the texts, for the crowded, dull, confusing appearance of the early books has entirely disappeared. (See Figure 3.)

Several of the texts before 1830 had margins only ¼ inch wide, but Pike's books, which were among the earliest, left ½ inch at the top and sides and ¾ inch at the bottom, the size of margins used almost universally in arithmetics today. Smiley's text, in 1828, and Talbott's, in 1833, enclosed the printed part of the page in a line border, with a fairly wide white margin around it. DeMorgan's *The Elements of Arithmetic*, in 1832, was the first to have very wide margins (⅝ inch at the top and sides and 1 inch at the bottom). None of the others left this much border around the printed page until the McLellan-Ames series in 1898.

The type used in the early arithmetics was very small. In the first 40 years of this study a number of texts had 8½ to 9 lines of type to the inch and the footnotes were so small that there sometimes were 12 lines to the inch. However, it did not take textbook writers long to get away from print so difficult to read. Only four of the books after 1842 had more than 6½ lines to the inch in the regular text and 8 lines to the inch in footnotes. Although a few arithmetics have been printed with only 5 or 5½ lines to the inch, all recent books have gone back to the 6-line size, first introduced by DeMorgan (1832), for all except the primary texts. Presumably, eye-movement studies have shown this to be the most satisfactory size for rapid and comprehensive reading.



(three eighths size)

Fig. 3. Pages from representative arithmetics, showing the comparative size of page, size and kind of type, and general attractiveness of the page

The cluttered appearance of the page in the older books was in large part due to the use of many different sizes and kinds of type, with the work crowded together in as small a space as possible. It was not uncommon to find content, explanations of examples, footnotes to pupil or teacher, and so forth, in different sizes of Roman type on the same page, with headings, "Ans.," mathematical terms, rules, etc.,

in italics varying in size according to the size of the Roman type they accompanied. Pike's 1788 book even printed the entire preface and table of contents as well as many emphasized phrases throughout the book in italics. The use of the old-fashioned "s," resembling an "f," did not add to the ease of reading in the first five books of the study.

As time went on, great improvement was made in simplifying the appearance of the page, although the use of different sizes and kinds of type was considered desirable through most of the nineteenth century. In 1877 Ray mentioned in his preface that "the different matter of the volume, the definition, the solution, or the rule, is at once clearly indicated by a difference of type" (34b:iii). However, italics lost in popularity; regular Roman type came into almost universal favor for content and notes; and bold-faced Roman type came into use for headings and replaced italics in many other places in texts published since 1910. Notes to teacher and pupil became less common, although they were used very frequently as late as 1875 and are still found now and then in textbooks. Also, when explanations of the rules and examples became a part of the content instead of a separate inserted "aid," they were set up in the same size and kind of type as the rest of the content of the book. Thus, much of the jumbled appearance of the page was done away with. Where examples and their solutions or explanations occurred in the older books, they were often set up in a double column and in some cases, where practice was offered in the use of certain rules, as many as three columns were used—this in a printed line of only about four inches in length! Seldom was enough space "wasted" to begin a new topic, or even a new chapter, on a new page; sometimes a line separated chapters, sometimes nothing.

The first of the Southworth series (1893-95) enclosed its rules in boxes about 1¼ inches wide. In this way the rules stood out clearly and the confusion of using a different kind of type was avoided. The idea of boxing was later extended to include definitions, tables, and drills in some of the most recent texts.

From about the middle of the nineteenth century and to well after the first of the twentieth century, almost all arithmetics numbered their "articles" or sections. These usually consisted of any new topic or rule introduced, a number being given to each rule, definition, and set of problems in some of the books, while in other books it was given only to each new topic. In only one or two books were these numbers always at the top of the page; usually they fell where they might, and often several numbers were found on each page. This over-organization of the subject enjoyed about a 60-year period of popularity, but when it went out of vogue it was dropped very quickly and, in appearance, the continuity of the page became much smoother.

Illustrations have added a great deal to the attractiveness of the page in the last 40 or 50 years. Before that time simple line drawings had been used to illustrate area or measurement and a few ornate headings had been included in some of the texts. But only two arithmetics before 1900, French's (1866) and White's (1870), had used pictures to make the book more interesting in appearance. It may have been that the value of illustrations had not yet been proved, and the cost of printing was still so high that it did not warrant the inclusion of unnecessary material. However, after it was found that pictures added

greatly to the attractiveness of textbooks, few authors failed to include them.

Color was introduced in 1928, by the *Modern Life Arithmetics*, and was found to add so much to the appeal of the text that it has been very popular ever since. Although only one color was used at first, pictures in the last three series analyzed were printed in several colors. The *Daily-Life Arithmetics* and the *Study Arithmetics*, two of the most recent texts, also adopted the new idea of allowing the illustrations to extend to the edge of the page, thus disregarding the time-honored custom of keeping regular margins throughout the book.

Surprisingly, the Johnson series (1900), the last half of the *Triangle Arithmetics* (1928), and the *Study Arithmetics* (1934-37) were the only ones to make use of photography. Most of the pictures used, however, were of buildings, business forms, adults in working situations, or children in school or on the playground. One objection to the use of photographs may be that it is difficult to get all the elements desired in an illustration and yet not have it appear to be posed. Drawings can often be made much more expressive, especially for younger children.

Thus, many factors have entered in to help make the inside of the book more attractive. Margins have been widened, print has been made larger, the open space on the page has been increased, fewer kinds of type have been used, and illustrations, both in black and white and in color, have been added. In fact, remarkable progress has been made in this phase of textbook construction.

Interest of Authors and Publishers in Physical Make-up of Book

In early times it seems that errors often crept into the texts in spite of great care and concern on the part of the author and publisher, who had no one but themselves, and perhaps a few friends, on whom to depend for checking the manuscript. Recognizing the possibility of typographical errors or inaccuracies of content, they often offered apologies to the public. Typical of such statements is one made in 1799:

> Perfect accuracy, in a work of this nature, can hardly be expected; errors of the press, or perhaps of the author, may have escaped correction. If any such are pointed out, it will be considered as a mark of friendship and favor by—
> The public's most humble and obedient Servant,
> Nathan Daboll. (8:vi)

Not since 1832, however, were any such apologies found in the texts, probably because most texts since that time have been put out by large publishing houses which have editorial and technical staffs made up of experts in their fields and whose duty it is to prevent errors from "slipping by."

As far back as 1849 publishers showed an interest in the appearance of their books. In Perkins' *An Elementary Arithmetic*, published in that year, the publishers stated that they took "pleasure in the *appearance* of the Book, which certainly *invites* the interest of the

scholar" (19:iv). It is hard today to look back on this book and agree wholeheartedly with the publishers. The book was small, covered in gray paper-covered cardboard and bound in black leather, with much small print on the cover, including the names of ten other books in the Perkins' series. Inside, the book was no more inviting than outside. The pages looked heavy and uninteresting, with small type, narrow margins, and no illustrations. When compared with other books of the times it was not outstanding and could hardly be said to warrant such praise as the publishers gave it.

In 1879 Olney, the author of *A Practical Arithmetic*, became quite verbose in praise of his book, stating that:

> The Author feels himself especially fortunate in securing publishers who bring out his books in the best style of the printer's art, and on the best material. The good taste of the stereotypers, the elegance of the engraved chapter-headings, the perfection of the press-work, the quality of the paper, the simple elegance and firmness of the binding, are mechanical features of no small moment in a textbook, and which in this cannot fail to challenge admiration. (36:5)

The book, however, was a medium-sized text covered with very dark green cloth-covered cardboard, printed and bound in black, with a conventional border at both the top and the bottom. The appearance of the page was an improvement over many of the texts of the day, in that there were good margins and plenty of open space between items, but there were still several different sizes and styles of type used. The book included 10 or 12 illustrations, all but two of which were very simple line drawings explaining measure or geometrical problems. The two exceptions were a group of apples illustrating the cutting of the whole into its parts, and a picture of a five-cent coin, with a statement as to its diameter and weight. Simple but rather artistic decorations were used in making cuts for each section heading throughout the book, but these were formal, and would not be particularly interesting to a child.

Few authors after this time praised the appearance of their texts. More recently they have drifted away entirely from such advertising in the preface, leaving the physical qualities of the book to speak for themselves or to be pointed out by those seeking textbook adoptions.

Chapter IV
Content

SINCE the content of arithmetic has usually been determined by what was considered practical, there have been very few important changes in the principles and processes included in textbooks in the field. Most of the changes that have taken place in content have been in the form of additions rather than of omissions, for it has been difficult to give up what has been tried and found desirable at some earlier date. Grade placement of material has probably accounted for the exclusion of more topics than has any other one thing, because many subjects included in early "complete" arithmetics have been advanced into the realm of higher mathematics and consequently out of the range of elementary school work.

Numbers were used long before arithmetic textbooks were written, even before formal instruction was offered in any phase of the subject. They filled a simple but real need for counting men or sheep or cattle. Then came a demand for more involved processes—for dividing, trading, selling, and, later, for handling more complicated business dealings. For more than two thousand years such processes as alligation, measurement, powers and roots, and progression have been taught and used. Measurement still receives an average of more than 3 per cent of space in arithmetics; powers and roots are found in almost every text; and alligation and progression have been dropped only within the last 50 years.

While many topics have been common to most arithmetics since 1790, the emphasis has changed from that of presenting content of practical value for the artisan to that of teaching the mathematics of everyday social usage. A few topics have disappeared entirely from texts; some that are still included appear in the more recent arithmetics under a new name; and a few headings today indicate a different type of content from that which they designated a century or more ago. On the whole, however, there was rather pronounced consensus of opinion among the textbook writers concerning the topics considered most important, and disagreement, where found, almost always concerned the minor topics.

Phases of Arithmetic Included

The relative emphasis given to the different topics by each textbook was measured in terms of percentage of space. A study of space allotted to each subject by the textbooks in this analysis (divided into five periods of 30 years each) was made by the Bureau of Cooperative Research and Field Service in 1942-43, and was reported in the Bulletin of the School of Education.[8] The findings of that space study have been used as a partial basis for the present more detailed analysis of content.

[8] See footnote, page 7.

Ten phases of arithmetic given the most space.—Since the arithmetics varied so greatly in number of pages (the total number of pages in all books of a series was considered as the number of pages in that text), percentage of space rather than number of pages given to a certain topic by each book or series was used to show true comparisons. The percentage of space given to the ten most common phases of arithmetic included in these texts is presented, by 30-year periods, in Table I.

TABLE I. PHASES OF ARITHMETIC GIVEN AN AVERAGE OF 3 PER CENT OR MORE OF SPACE IN TEXTBOOKS OF AT LEAST FOUR OF THE FIVE 30-YEAR PERIODS

Phase	To 1820	1821-1850	1851-1880	1881-1910	1911-1940	All books
Whole numbers	5.78	17.65	16.22	24.19	30.21	22.35
Percentage	14.81	8.59	13.41	12.19	11.25	11.99
Fractions	4.89	11.75	11.94	11.57	11.08	11.43
Division	4.95	9.69	10.81	5.16	8.84	9.32
Multiplication	5.99	8.88	6.93	6.02	7.36	6.30
Compound denominate numbers	13.20	10.84	9.02	5.68	1.11	6.19
Geometry	3.66	3.57	5.18	6.59	4.27	5.22
Addition	3.58	6.24	4.97	4.47	7.07	4.98
Decimals	4.75	4.94	5.66	4.64	5.02	4.95
Subtraction	2.77	4.46	4.49	3.18	5.63	3.86

Stress on work dealing with whole numbers is evident in Table I, as this topic was given almost twice as much space as any other. Such a situation is not surprising, since a knowledge of whole numbers is so necessary to an understanding of other phases of arithmetic. The value of a clear understanding of whole numbers has apparently been felt increasingly through the years, for at the end of the 150 years this topic was being given more than five times as much space as at the beginning. This increase in emphasis may be accounted for by the fact that in the early days the student was plunged immediately into the handling of large numbers rather than simple ones, and, soon after, he studied fractions and decimals. Since his training in arithmetic consisted almost entirely of the memorizing of rules and definitions and the copying of "sums," a basic understanding of the work did not seem necessary. Recently, however, educators have attempted to give pupils a "feel" for numbers before taking up more complicated processes in arithmetic, and they have therefore given a great deal of space to building up a more thorough groundwork in simple whole numbers.

Percentage and fractions were each given an average of 11+ per cent of space in these arithmetics. The emphasis given to percentage has remained fairly constant through the years, although it was stressed more in the first period than in any later period. With the coming of the inductive method and the attempt to simplify texts, percentage dropped considerably in emphasis, but it was not long

before it was back in textbooks almost as strong as ever but in a more general form. While there has recently been a slight decrease in the amount of space accorded the topic, it is still considered a subject which pupils should thoroughly understand.

The amount of space given to fractions has remained about the same for the last 100 years or more. In the first period of this study less than 5 per cent of space was given to fractions, but since 1821 between 11.08 and 11.94 per cent has been accorded the subject in every period. Fractions were included in every arithmetic analyzed, even in those early books designed especially for children. In such texts they were sometimes written out to make them more understandable. Colburn, for example, expressed fractional terms as "2 sevenths," "4 ninths," and so forth. All but two of the texts before 1840 referred to fractions as "vulgar" fractions. DeMorgan (1832) dropped the word "vulgar" and Adams (1827) substituted "common." Since about 1850 this phase of arithmetic has been referred to either as "fractions" or as "common fractions," the former term being used in all arithmetics published since 1928.

Space listed as given to the four fundamental processes—addition, subtraction, multiplication, and division—includes all space, whether in whole numbers, fractions, or decimals, given to these operations. The earlier arithmetics gave multiplication and division two or three times as much space as addition and subtraction. Then, until about 1850, the emphasis shifted to division alone. Since that time addition, multiplication, and division have been given approximately the same amount of space and subtraction alone has lagged behind. The trend seems to be toward considering the four of more equal importance, as the space given to subtraction is increasing steadily.

Compound denominate numbers, a topic which included the mathematical handling of every type of denominate numbers found in the texts, was consistently given less space in each succeeding period. One reason for this is that an understanding of exchange values, in connection with both foreign and domestic exchange, was essential when the first arithmetics in this study were published. Another reason probably is that recent scientific studies of the uses of arithmetic have shown that many of the measures taught in early arithmetics were not needed by the average person. When these were dropped from textbooks, instruction in their manipulation was not included. Much of the work in compound denominate numbers was known as "reduction," or the bringing of "great names into small" and of "small names into great" (8:63) without altering their value. Textbooks often included whole chapters on reduction, the subject being allotted more than 10 per cent of space in the first period. By the last period, however, it was stressed very little.

The emphasis given to the teaching of geometry and of decimals has remained about the same through the years so far as amount of space accorded them is concerned. Geometry received greatest stress between 1850 and 1910, but there was less than 1 per cent more space given the topic in the last period of the study than in

the first. The space given to decimals was even more consistent—in all five periods studied there was only 1 per cent of difference between the highest amount and the lowest. The form in which decimals were written was not standardized until after 1850. Only four books before that date wrote them as we do today and three others used the decimal point in Federal money but not in other decimals. Neither Cocker nor Colburn wrote decimals as such. Half of the other books in the first two periods used a comma instead of a "separatrix," as it was called. Ostrander used a space where the decimal point should be, DeMorgan and Perkins used a period placed above the line of print, and Adams used what appeared to be a single left-hand quotation mark. However, after 1850 the use of the decimal point as it is written today had become so well established that it was accepted by all textbook writers.

Other phases of arithmetic included.—A number of other topics were found in almost every arithmetic throughout the 150 years, but none of them was given as much as 3 per cent of space in at least four of the five 30-year periods. Measures, numeration and notation, ratio and proportion, powers and roots, and proof were at least touched upon by almost all authors, and the first four of these were given considerable space.

Measures were given more space in each succeeding period, being accorded less than 3 per cent in the first two periods and more than 10 per cent in the last. In the early arithmetics a large proportion of the space allotted to measures was taken up with all kinds of tables of measures, sometimes including such unusual things as diamond weight, chain measure, and cheese and butter measure. These are presented more in detail in connection with the discussion of tables. More recent textbooks included fewer and briefer tables of measures but more explanatory material and problems in the field.

Numeration and notation (the reading and writing of numbers) also increased but was not given as much as an average of 5 per cent of space in the books of any of the periods. In the early texts, charts were usually used to present the organization of the numbering system. Numbers up to 25 places or more were sometimes included, but little explanation accompanied the chart, which apparently was given purely for reference purposes. A similar kind of chart was also used in teaching the reading and writing of decimals. Few recent texts have included numeration charts, and those given are much simpler than the early ones.

Ratio and proportion and powers and roots, on the other hand, decreased in importance, ratio and proportion dropping steadily from 9.00 per cent of space in the first period to only .83 per cent in the last. This drop may have been due, in some degree, to the fact that some phases of the subject have been advanced to higher mathematics and omitted from the elementary texts. One reason for the decrease in amount of space given to powers and roots is probably that in early texts powers up to nine were included. The higher powers, however, were dropped from texts after studies of the practicality of

topics were made. Even cube root was left out of almost all texts after 1900 and much less space was given to square root.

Of the subjects which have been advanced into higher mathematics, algebra and physics were included in books scattered throughout the study. Algebra was found in few of the early books, but it usually received rather extensive treatment when included. Later, grading of subject matter in mathematics placed the subject in the higher brackets. It has been touched upon in the majority of the texts since 1900, but only the most elementary phases of the subject have been discussed. Physics was included in only 11 of the 59 books, but these were divided almost evenly among the five periods. In recent books only very elementary principles have been included, while in the early books the subject was taken up rather thoroughly from a mathematical point of view. Logarithms, conic sections, and trigonometry were recognized early as belonging in advanced phases of mathematics, and none of these was found in textbooks after 1801, although the fundamentals are still taught indirectly. Astronomy and chronology also have been dropped from elementary texts, though they were discussed at some length in Pike's 1788 text and were mentioned in one or two of the other very early arithmetics.

Alligation and progression dropped out of textbooks about 1900, but are in many texts still touched upon under a different classification. For example, almost all arithmetics have included the principles involved in alligation, "that Rule whereby we resolve Questions concerning the mixing of several Simples, or Commodities into one compound Quantity" (4:276). Early texts set it up as a main topic, but it has been a separate topic in only two texts since 1878. In more recent arithmetics it has been included as a part of the discussion of ratio and proportion.

Position, or the rule of false, was dropped before 1835; and permutation, "which shows how many different ways the order or position of any given number of things may be varied" (10:134), an important rule in early days, was not found in texts published after 1865.

Duodecimals were greatly emphasized in the early years of the study. Walkingame (1751) gave an entire part (Part IV) of his text to the subject. S. Pike (1811) described duodecimals as "fractions of a foot, or of an inch, or parts of an inch, &c. having 12 for their denominator. The denominations are, foot, inch, second, third, and fourth" (6:188). They are no longer taught except for a remnant of the system found in certain measurements. It was difficult for educators to accept the decimal system, however, because it seemed so logical to use a unit divisible by more numbers than was 10. Until as late as 1865 most texts included this topic even though they admitted the value of decimals by including them also. Duodecimals were not finally dropped from arithmetics until the 1880's.

Practice, "a concise method of finding the value of any quantity of goods, by the given price of one integer" (13:78), was a formula, or equation, which received its name from its frequent use in business. This of course takes it entirely out of the realm of what would

be considered "practice" in more modern texts, for the term has come to be used to designate a rather informal type of drill. Practice as taught in old textbooks lost importance with the introduction of Federal money and was dropped entirely soon after the use of U.S. currency became standard.

On the whole, great uniformity was evident in the amount of emphasis given to the principal phases of arithmetic by the different authors. Eight of the ten phases given most space in the texts were included in every one of the 59 arithmetics studied, and one of the others, percentage, was found in all except one book. While geometry was not discussed in some of the early texts, it has been included in all arithmetics of the last 100 years.

In almost all periods the greater stress was given to the more general principles and processes—to the four fundamentals, whole numbers, fractions, decimals, and percentage—one or more of which are necessary in every problem involving number. Some of these phases gained or lost in favor during the 150 years, but most of them remained remarkably constant. Processes of less importance were given less space and were included in fewer texts, while those of little use today were either dropped entirely or included as a part of some more important topic.

Order in which phases were taken up.—There was unusual consistency in the order in which topics were presented as well as in the emphasis given them.

At the time the early texts of this study were written there were no scientific findings to guide the authors in determining this order and most arithmetics followed a plan similar to that of Pike (1788), who stated that he had "arranged the Work in such order as appeared to me the most regular and natural" (3:5).

In general, numeration was included first, usually on the beginning page. In the early texts a full discussion of the subject was given immediately, but after about 1890 most books included numeration of whole numbers at the first of the book, numeration of fractions at the opening of the discussion of fractions, numeration of decimals when decimals were first presented, and so forth.

As a rule, the four fundamentals were taken up next. Only two of the 59 texts took these up in any order other than the traditional addition, subtraction, multiplication, and division. Daboll (1799) followed addition by division, then multiplication, and lastly, subtraction, while Smiley (1828) introduced multiplication second and subtraction third. Smiley followed addition by multiplication on the theory that it is a series of additions and that division is a series of subtractions, but no good reason could be found for the order used by Daboll.

Almost all authors agreed that whole numbers should be taught before fractions and fractions before decimals. Ostrander (1829), however, discussed decimals before fractions because, as he said, the computational aspects of whole numbers and decimals were the same except for the placing of the decimal point. Other authors explained that they had included decimals or "decimal fractions," as some called them, after fractions because they were truly a form of fraction

with ten or a multiple of ten as a denominator. Several texts, the earliest published near the beginning of the nineteenth century and the most recent published more than a hundred years later, placed the discussion of Federal money, or U.S. currency, before that of decimals. While this plan may not have led to a thorough understanding of the meaning of decimals, it did provide a rather natural introduction to their use.

In most early texts the Rule of Three was presented immediately after the discussion of these more common topics. This rule was used in solving problems in alligation, position, proportion, fellowship, progression, etc. and in certain kinds of business practice. Most textbook writers considered that the Rule of Three, with its variations, was as complicated a process as anyone needed in practical arithmetic and many textbooks included no work more advanced than the various topics in which this rule was used.

On the whole, the order in which topics were taken up varied in no important details through the years. At first one topic followed another merely as a new topic, each being given the same emphasis so far as headings were concerned. Later they were grouped under main headings, but the same general order of topics persisted even when methods of teaching changed radically.

Types of Business Transactions Included

While business practices are constantly changing in minor respects, the fundamental principles back of them are essentially the same as they have been for hundreds of years. The figuring of profit and loss, for example, is found in a great number of transactions, and the paying of interest or commission has been a part of business procedure since long before Columbus came to America. Because a knowledge of such subjects as these is necessary to almost everyone in his personal dealings, if not in his work, all textbooks have included instruction in business arithmetic. In solving problems in this field, the pupil must not only know how to handle abstract numbers, but he must have an understanding of the meaning of the problem so that he will recognize the principles involved and know the processes to be used.

Types of business transactions included in at least two thirds of the texts.—Interest and profit and loss were found in every textbook analyzed, though they were not always listed under separate headings. Discount, insurance, commission, and U.S. currency were also found in at least 50 of the 59 texts studied.

In the early books, interest was always divided into simple interest and compound interest with often a number of "short and easy rules to calculate interest" (14:134) included also. Problems usually were more or less abstract, testing only the ability to manipulate figures correctly, or were based on big business transactions. Later, interest was usually taught in connection with small savings accounts or other matters with which the average person was acquainted.

The teaching of profit and loss has varied but little through the years and the emphasis given the topic has remained fairly constant.

The topic of discount in this study includes various kinds of discount. In the first two or three 30-year periods, discount and present worth (the figuring of the value of a debt at a given date before it is due) was included in almost all texts. Tare and trett, a form of discount allowing deductions for the weight of the carrier and for loss sustained in transportation of merchandise, was also a favorite subject in early arithmetics. The principle is still found in texts, but not under that heading. Commercial discount was included in arithmetics early in the nineteenth century, but bank discount was not found until 1848. In more recent texts the emphasis has been placed on the kinds of discount most often met with by the average man—cash discount, trade discount, and the discount given at special sales.

Insurance of different kinds was taken up in the majority of the texts. In the early arithmetics problems dealt almost exclusively with property insurance, especially that of cargo on a ship. More recent texts, however, included several kinds of insurance—fire, theft, life, etc. The topic of annuities also is included in this analysis under the heading of insurance. In early books annuities was given a great deal of space, with the pupil being asked to figure the values of annuities under certain stated conditions. The subject was found listed as a separate topic in only two books after 1877. Evidence indicates that, in more modern arithmetics, the principles involved in the annuity have been presented in problems concerning endowment policies, under life insurance.

Commission, or "factorage" as it was called by Pike in his 1788 arithmetic, was found in every text after 1825 and in more than half the texts published before that date.

The content of problems concerning U.S. currency has probably changed more than that of most problem topics. At first "decimal" money was treated as a novelty, as a new kind of currency interesting to figure with but impractical and very unlikely to replace Sterling money. Although U.S. currency was adopted by Congress in 1786, it was not sufficiently accepted in business practice to be included as an essential part of arithmetics until much later. During these early years there was no standard form for writing values in U.S. currency and tables giving "the value of a dollar in each of the United States" (12:120) were given in the texts. These, however, were dropped soon after standard currency was generally adopted. Since the use of the decimal point in expressing dollars and cents was not yet fully established, texts often avoided the issue by using "D." for dollars and "cts." for cents.

Thompson's text (1825) was the first to recognize the fact that American money was supplanting English currency. After this a number of texts, designed especially to prepare pupils to do business in the United States, emphasized the currency of this country.

S. Pike's text (1811) was the first in this study to use U.S. currency rather generally, and Hassler's *Elements of Arithmetic* (1826) was the first not to use English currency in any of its problems involving money. All earlier arithmetics presented most of their ex-

amples and at least part of their problems in English money. Colenso's book (1864) was the only text after the middle of the century to contain any extensive treatment of English currency, but this text was published by the London branch of Longmans, Green and Company and it was therefore logical that its problems dealt with English money. Four books printed after 1865, one of them as recently as 1878, still gave some of their problems in Sterling money.

Bills and accounts, promissory notes, imports and exports, taxes, foreign and domestic exchange, and equation of payments were all found in more than two thirds of the texts.

Promissory notes were found in only six texts published before 1848, but were included in all arithmetics after that date.

The saying that taxes are always with us is true in these textbooks, for the topic has been included in all texts published in the last 75 years. The surprising thing is that instructional information concerning taxes accompanied the presentation of the topic in many of the arithmetics. Even as far back as Pike's *New and Complete System of Arithmetic*, in 1788, the first paragraph of a footnote on taxes read:

> It may not be amiss to shew the general method of assessing town or parish taxes. First, then, an inventory of the value of all the estates, both real and personal, and the number of polls, for which each person is rateable, must be taken in separate columns: The most concise way is then to make the total value of the inventory the first term, the tax to be assessed, the second, and £.1 the third, and the quotient will shew the value on the pound: 2dly, Make a table, by multiplying the value on the pound by 1, 2, 3, 4, &c.—3dly, From the inventory take the real and personal estates of each man, and find them separately in the table, which will shew you each man's proportional share of the tax for real and personal estates. (3:133)

In recent texts the discussion has more often concerned the uses to which tax money is put—the percentage spent for education, sanitation, administration, etc.

Foreign and domestic exchange was also included in most of the texts studied, but the topic was given more space in early arithmetics than in later ones. At first, the term "exchange" was used almost exclusively to mean "the process of making payments at distant places by the remittance of drafts, instead of money" (37b:253), and a study of foreign exchange included the learning of the values of coins of different countries and the rates of exchange. Some of the older texts included tables of coins and their values in Denmark, England, France, Ireland, Italy, Portugal, Russia, Spain, and even India, China, and other countries. The student was expected to memorize all of these as well as to know how to use them in solving problems.

Problems involving foreign or domestic exchange in early texts dealt almost exclusively with rate of exchange, but in later periods the term was expanded to mean the transfer of money by almost any means other than actual cash—drafts, money orders by mail or wire, checks, and so forth. As emphasis shifted to exchange of this type, fewer texts included tables of foreign currency values. In the more

recent arithmetics, mention is made of the fact that there are differences in values, but the topic is given little space, as the average person needs only to know that such a situation exists.

A discussion of bills and accounts was included in almost every arithmetic published after 1860. In early texts they were often referred to as "Merchants' Bills" (34b:83) or as "bills of parcels," a listing of the separate "parcels of goods" included on the bill. Forms in which bills were made up were often presented also in connection with the study of this topic.

Equation of payments, "the process of finding an equitable time for the payment of several debts due at different times, without interest" (37b:261) was found in 42 of the arithmetics, but has not been included under that heading since 1913 in any of the texts studied.

Imports and exports as a topic includes space given to duties and customs, since these two topics are so closely related. While problems dealing with imports and exports were found in all periods, duties and customs did not enter the arithmetics until after 1820. This situation probably came about as a result of the increased emphasis on such matters after the country gained its independence.

Other business transactions included.—Barter and fellowship were found in only a few of the texts, but are included in this discussion because they were a part of almost all early arithmetics. Barter, however, was not included in the texts after 1834, and fellowship was dropped by the middle of the century.

Problems in barter often involved incidents in which farmers exchanged crops or lumber for cloth or sugar or wine. Since the adoption of an acceptable monetary system, barter has not been a common practice in business dealings, and for this reason the topic is no longer included in textbooks.

Fellowship as a topic also showed evidence of dropping out of arithmetics by 1850, but a change in terminology was responsible for this seeming omission, for the topic has continued to be included in texts under the heading "partnership." Even under this heading, however, the topic again disappeared from texts about 1913.

Bookkeeping practices and forms were included in arithmetics from the beginning of this study. Gough, in 1801, included an entire section in his book entitled "Mercantile Arithmetick" and S. Pike, in 1811, also emphasized the subject. Presentation of various phases of bookkeeping was considered a very helpful and "practical" type of information, since the textbooks were written primarily for persons needing such facts. The technical side of the subject was gradually dropped from elementary texts and placed in commercial arithmetics, but the more common and useful phase, that of teaching the understanding of bills, receipts, and so forth, is still found in textbooks today.

Many topics were included in the arithmetics that have not been mentioned here. Stocks and bonds, building association business, mortgages, contracts, bankruptcy, and a number of other topics were found in old as well as new texts. However, there apparently was little agreement among textbook authors as to the emphasis to be given such subjects.

The order in which the topics have been taken up has not varied much. Interest, profit and loss, and discount have usually been among the first business proceedings taken up, and those included less often have usually been found in the latter part of the book. Slight trends have been noticeable in the grouping of topics, however. In early days all types of problems which could be solved by the Rule of Three were found under that rule, regardless of their relationship to each other either in principle or practice. Later, topics were grouped more nearly according to principle, with all those dealing with percentage, for example, grouped together. More recently they have shown a tendency to be listed according to practice, with, for instance, matters in which banks and banking are concerned being taken up in closely related sections of the book. While there is great overlapping in the use of these methods of grouping subjects, the trends are worthy of note.

The greatest difference between the business arithmetic included in modern texts and that in arithmetics of 150 years ago is in the type of subject matter used to illustrate the principles. Information included in early texts was that needed by business men in their dealings with other business men, while in the late books of the period the emphasis was on the type of transactions carried on by the man on the street.

Arithmetic of Daily-Life Activities

Many of the everyday experiences of people involve the use of number, but until comparatively recent times authors of arithmetics did not seem to recognize these as important enough to be included in textbooks. They often were things learned outside of school—things that perhaps seemed too elementary for the classroom. These are the topics included in what are here termed the arithmetic of daily-life activities.

Although the less involved phases of such topics as profit and loss, taxes, insurance, etc. could probably be correctly classified as daily-life activities, they have not been included as such because in the textbooks analyzed they were almost invariably presented as business transactions. It has only been within the last few years that emphasis has been shifted to the more personal phases of these activities.

None of the topics included in this classification was mentioned in any of the texts published during about the first 60 years covered by this analysis. Checks were the first of the subjects mentioned, but they were merely defined by Eaton (1863) and Dean (1865). A little later, Hagar (1871) and Raub (1877) added to the definition a description of the form in which checks were written, but none of these authors included any problems on the subject. Not until the 1880's was the subject of personal checking and savings accounts introduced into textbooks as a basis for problem solving.

Standard time was mentioned first in Dean's text but was not discussed again in these arithmetics until the Indiana Educational series (1886). After that, a number of texts included the subject.

The telling of time, a more useful and necessary phase of the understanding of time, appeared first in the Woodward series (1899), although pictures of clocks (used only as headings for the discussion of tables of time) were found in texts published about 30 years earlier than this.

The telling of temperatures and the understanding of their meanings were found in practically every text of the twentieth century, but only one book before that date, Dean's *The Intermediate Arithmetic*, in 1865, included the topic. Apparently the attempt of Dean to present material seemingly very elementary and somewhat removed from the principles of arithmetic taught in school was taken up by very few other writers until several decades later.

The making of change, another activity common to the life of a child even of elementary school age, was first introduced into texts in the Southworth series (1893-95). Many arithmetics before this time had contained questions concerning the amount of change due a customer, but the topic "making change" refers here to counting out the change properly, as it is done in stores.

The introduction of these topics came at about the same time as the making and reading of graphs and scale drawings, and the sending of mail.

The sending of telegrams and the reading of gas, electricity, and water meters were first presented as topics in the *Everyday Arithmetic* series (1915), the second of these subjects being included in all but one of the texts published after that date. The making of budgets, another topic found in all recent arithmetics, was first introduced by the *School Arithmetics* (1919).

In 1928 the *Triangle Arithmetics* added the reading of maps, a topic which was included in all subsequent textbooks.

Apparently an attempt was made by most of the authors to introduce these daily-life activities in the order in which they would most likely be met in life. That is, telling time and making change were usually found in the primary texts, handling mail and personal bank accounts in the intermediate books, and reading maps and meters in the upper grade texts.

A number of other daily-life activities were included in some of the more recent arithmetics, but none of them was found in more than two or three texts. No doubt the need for some of these is just beginning to be felt, but they may become a more or less essential part of the textbooks of the future. The trends seem to be definitely toward making arithmetic a living subject, with a view to teaching the child the kind of arithmetic he uses every day, and presenting it when he first feels a need for it.

Forms of Presentation

The many phases of arithmetic included and the principles and processes taught in these textbooks were presented in a variety of forms. Rules, examples, problems, and so forth were among the types of material used in explaining a new topic and giving practice in its application. Principal among the means used for presenting arith-

metic are those given in Table II. Here again percentages are used in order to give a comparative picture of the texts of the different periods. These figures, also, are taken from the five earlier studies.

Principal means of presenting arithmetic.—According to Table II, more than 25 per cent of the space in texts was taken up with written problems. This classification included all questions and concrete problems but did not take in abstract problems or pure drill. All phases of arithmetic and all principles and processes presented in the form of written problems were included as well as written problems used as examples, as practice work on a new topic, as review, and as test material.

TABLE II. FORMS OF PRESENTATION USED IN ALL ARITHMETIC TEXTBOOKS

Form	To 1820	1821-1850	1851-1880	1881-1910	1911-1940	All books
Written problems	19.41	24.69	26.72	22.74	39.01	25.70
Drill problems	11.51	13.81	17.44	29.43	22.83	21.19
Examples	30.75	24.42	22.61	7.58	16.58	11.78
Rules	18.39	12.46	10.09	2.01	1.03	6.35
Definitions	4.10	7.27	10.10	5.98	.57	4.88
Tables	7.07	6.22	5.77	1.84	2.56	3.03

From the beginning of the period studied, the written-problem form of presenting arithmetic principles and processes was given more and more space in texts, partly, perhaps, because few problems other than examples were included in early arithmetics and partly because there was greater interest in the nineteenth century in developing and testing thought power, an ability believed to be best increased through the use of written problems. The character and wording of the problems in early texts were so unusual and the changes that have taken place were so interesting that an entire chapter of this study (Chapter VI) has been given to them.

Drill problems gained favor steadily until 1910, but in recent texts were not found as often as at the peak of their popularity, 1881 to 1910, when it was thought skill and accuracy could be obtained by drill only. Included under the heading of drill were all abstract problems designed to give exercise or practice in certain fundamentals as well as all problems listed as drill. Although addition, subtraction, multiplication, and division tables had been included in textbooks for a century or more, actual drill tables were not found until about 1900. These were often called practice tables. In books of the last 30-year period, an attempt was made to give more space to drill on the number combinations most frequently used and to omit some of the involved problems found in early texts.

Together, written and drill problems comprised almost half of the space in textbooks. The only other form of presentation occupying as much as 10 per cent of space was that of examples. Classified in this group were all examples, whether given to illustrate rules or

some other phase of content, as well as all explanations that accompanied them. Analysis, also, was included here because it was used as a form of explanation for the various processes involved in certain problems.

More than 30 per cent of space was given to examples in texts of the first 30-year period studied. This large amount of space is accounted for to a great extent by the fact that the explanations of examples were often given in great detail, each minor step in the analysis and procedure being included and discussed. At least one example accompanied almost every rule, and sometimes each slight variation of the rule was illustrated also.

The amount of space given to examples dropped quickly until about 1910, when such material was included only for the purpose of clarifying the more involved phases of new topics. Apparently the decrease in space given to the subject did not prove satisfactory, perhaps because it had been such a decided reduction, and in the last period of the study the space accorded examples was greatly increased.

Not much more than half as much space was given to rules as to examples throughout the 150 years, although rules were found in every textbook analyzed.

Nearly one fifth of the early textbooks was taken up with the statement of rules—rules for solving each new type of problem encountered, then rules for handling exceptions that might arise in cases of this type, and finally, rules for proving the correctness of the answers obtained. Among the rules, the Rule of Three was considered by far the most important, and was given a great deal of space, since it could be used in one way or another in solving problems of many different kinds.

After the publication of Colburn's *Intellectual Arithmetic*, in 1821, rules lost rapidly in favor. The inductive method, which led the pupil from the simple to the more complex until he discovered the rule for himself, became very popular, and rules were no longer stated at the beginning of every new topic. Only about two thirds as much space was given to rules between 1821 and 1850 as before that date. Ostrander, in 1829, stated that in his text "unimportant rules . . . have been discarded, to make room for matter more pleasant and useful" (13:3), and Chase, in 1848, claimed that "many of the rules which are usually given, are dispensed with, by teaching various applications of which each rule is susceptible" (18:ix).

By the fourth period of this study (1881-1910) the space allotted to rules had dropped to only 2 per cent, and in the last 30 years it was only about half that much. The idea that one is dependent upon a series of stated rules for an understanding of new material has been proved false, and recent authors are presenting arithmetic on the theory that understanding of principles and processes is more important than knowledge of rules.

Definitions became especially popular in the middle period of this study, but in recent books have been given even less space than rules. As in the case of examples, the space recorded as given to definitions also includes explanations of the definitions.

In most of the texts definitions were found where their need was first felt—accompanying a new rule with an unfamiliar vocabulary or in connection with new topics which involved the use of "new" mathematical terms. In a few texts, however, they were grouped. Milne's arithmetic (1877), one of the first to group them, included a page or two of definitions here and there through the book, and a few other texts in the late 1800's gave lists of definitions. Later this became a more common practice, the lists being referred to usually as "vocabulary" and being found most often in the back of the text.

Another form of presentation which has lost in popularity, as judged by the space accorded it, is tables. As in the case of examples, space allotted to tables constantly decreased through the first 120 years of the study, but in the last 30 years it has increased slightly. Tables were given much less space than examples throughout the years, however.

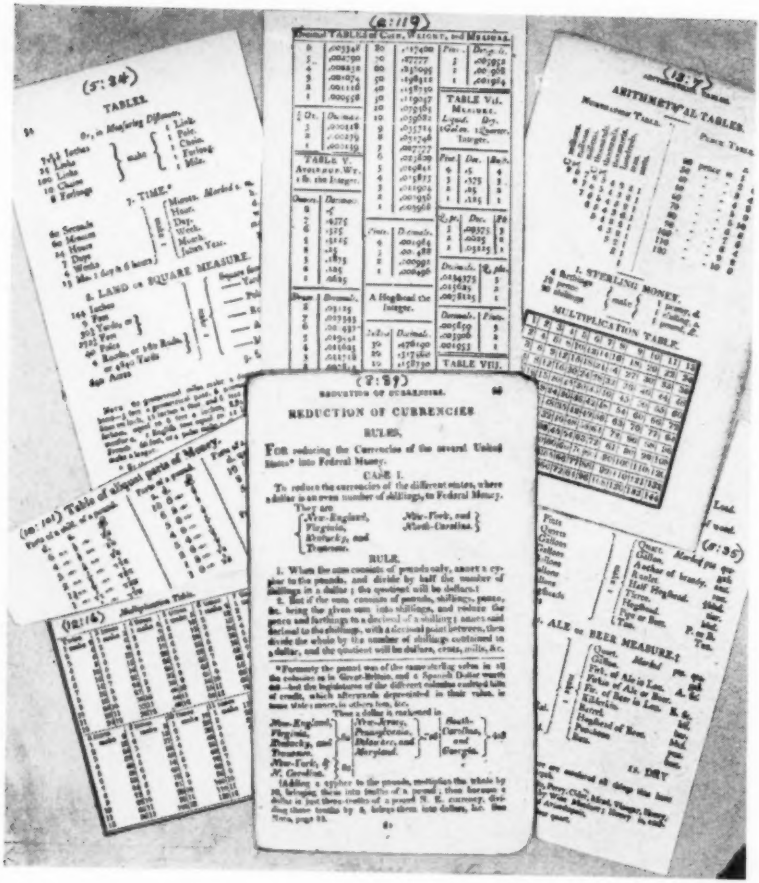
Classified as tables in this study are all tables of combinations in the four fundamentals, fractions, and powers and roots; all tables of interest and annuity values at different dates, and tables showing the number of days between certain dates; all tables of measures of various kinds, including currencies of different countries; and miscellaneous tables. (See Figure 4.)

While there was only a slight reduction in the amount of space given to tables of whole numbers and fractions, space devoted to powers and roots amounted to very little in the later texts. While in early arithmetics tables of powers and roots up to and including the ninth power were found, they disappeared entirely in the last part of the nineteenth century and the early part of the twentieth. Since then the simpler and more useful ones, tables of squares and square roots, have appeared again in a few texts.

A table showing the number of days between one date and another was very commonly found in early texts. This was used in figuring interest, annuities, and so forth. With the coming of "interest" tables, the table of days was dropped because it proved less usable than the interest tables.

Miscellaneous tables were almost never included in the more recent texts, and not nearly so much space was given to tables of measures as in the early texts. Three of the main reasons for this are that measures used only in specialized fields have been dropped from arithmetics; that the "extremes" not commonly used in tables have been omitted; and that tables of currencies, which used to be so necessary in business arithmetic, are seldom needed today and therefore have not been included in textbooks.

Pike's 1788 text, Guthrie's *American Schoolmaster's Assistant* (1810), Daboll's *Schoolmaster's Assistant* (1820), and Ostrander's *Elements of Numbers* (1829) each included six or seven pages of measures near the beginning of the book. Most of the early arithmetics and all of the texts in the last 30-year period placed these tables at the end of the book, sometimes giving several pages to the subject. During the middle years of the study such tables were scat-



(slightly less than one half size)

Fig. 4. Examples of tables found in arithmetics published before 1830

tered throughout the texts, falling wherever the author wished to take up the subject.

Of the six forms of presentation mentioned in Table II, four were used by every textbook and the other two were found in all but one of the arithmetics.

Other forms of presentation.—Other important types of material were illustrations, units of instruction, tests and drills, notes, answers, and explanations of symbols.

With the exception of the engraving of Edward Cocker, opposite the title page of his textbook, the first illustrations in arithmetics

were in the form of line drawings in connection with problems of area or plane geometry. Pike's 1788 text was the only one in the first period to contain illustrations, and it gave so little space to them that they could scarcely be said to be a part of the content of the book. Guthrie's *American Schoolmaster's Assistant* (1810) and Thompson's *Youth's Assistant in Practical Arithmetic* (1825) were the only other texts before 1835 to include such material. After that date the majority of arithmetics used simple line drawings or illustrations of business forms to help clarify their problems.

Although French gave almost 15 per cent of space to pictures in his *First Lessons* (1875), the majority of texts published before 1904 gave them only about 1 per cent, and even as late as 1879 arithmetics were found with no illustrations at all. Beginning with the Smith series however, most of the texts contained many pictures, some of the more recent books, giving them 5 to 10 per cent of space. In all series including illustrations of interest from more than a purely instructional standpoint, much more space was given to this type of material in the books designed for the lower grades.

The unit of instruction, or the grouping of problems around a central theme other than the development of a new principle or process, is a form of presentation that was not accepted to any extent until well into the twentieth century. French, in 1875, introduced many of his problems in this form, usually grouped around a picture which furnished the story background for the problems, but his plan was not adopted by others for many years. In recent texts units of instruction as a type of content have taken up much space. In fact, in one or two most of the book has been centered around family or community activities, with the teaching of the new processes as well as the problems being based on these activities. This method has gained its popularity through the fact that it is believed to give the text more of a storybook character, to make problems seem a part of daily experiences, and thus to make the study of arithmetic more attractive to the child.

Notes as a form of presentation of subject matter included not only those scattered through the pages of the text, but also those grouped together and given on a page or two at the beginning or end of the book. Colburn was one who included many notes on method in the back of his text, and other writers, among them recent authors, have devoted two or three pages to a section with some such title as "Suggestions to Teacher."

For purposes of this study, no distinction was made between notes to teacher and notes to students, as they often were addressed to no one in particular and could have been used by either teachers or pupils. Some authors used them to provide informational background as an aid in the understanding of a problem; others merely made suggestions in notes as to method or process to be used.

Notes became more common after the middle of the nineteenth century, because authors at that time had become very conscious of the lack of teacher preparation and used this means to suggest a

desirable method of conducting the lesson; in no period of this study, however, was more than 2 or 3 per cent of space given to notes.

In most arithmetics notes appeared on the page where needed, rather than at the front or back of the text, but were usually in smaller type than the rest of the content. They were sometimes at the bottom of the page but were more often placed immediately following the problem or topic with which they dealt. If given a designation, they were variously called "remarks," "notes," "suggestions," and, in the more recent books, "reminders" or "helpers." Baker, in his text in 1878, used the scholarly term "scholium," in keeping with the rest of his text, which presented rules and principles as theorems or corollaries. The space allotted to this type of content decreased considerably in the last period of the study, partly because the notes were shorter and partly because they appeared less frequently. They are still considered a more or less essential part of an arithmetic textbook, however.

Answers to problems were included in a majority of the texts. Only three arithmetics before 1895 (Davies' in 1852, Felter's in 1862, and Mecutchen and Sayre's in 1877) omitted them. A few others included only part of the answers, and Watson *et al.* (1805) and Chase (1848) published two editions, one with and one without answers.

Chase's *Common School Arithmetic* was the first to group the answers in the back of the book, all earlier texts having given each answer immediately following the problem. For this reason, until answers were grouped it was very difficult to measure accurately the amount of space given to them. It is assumed that in the early books the space allotment for this type of material was the same as in the later ones, with the exception that it would vary in the same proportion as the space given to written problems varied. Thus, there would be less space given to answers in the first period because there was less given to written problems.

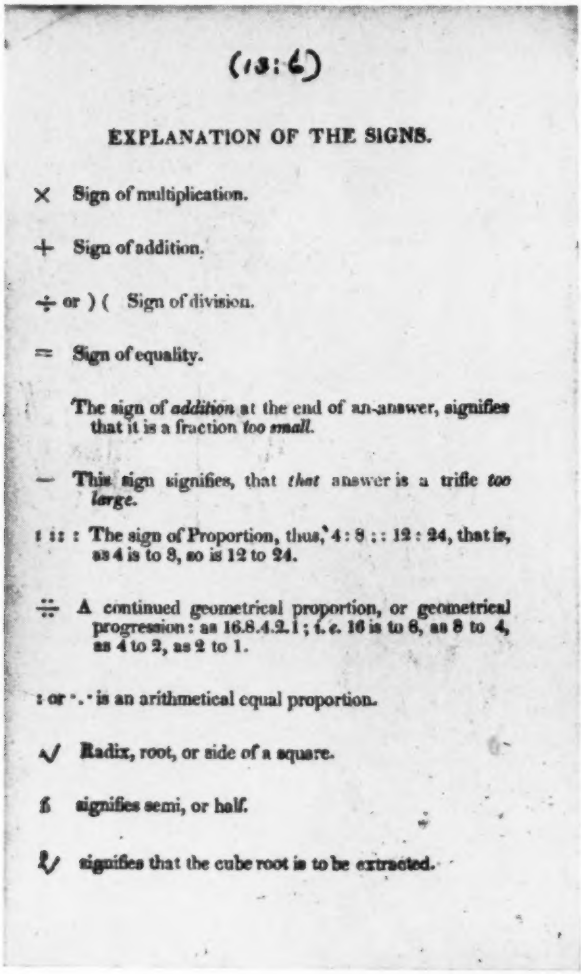
When included, answers comprised about 10 per cent of space in a text. The average amount of space given to this material in the second and third periods was approximately as much as this, but, as has been said, in the first period there were fewer problems and therefore fewer answers, and in the last two periods nearly half the books omitted answers altogether, which would of course cut the average considerably. The practice of including answers is still in favor with some educators, as four of the ten texts analyzed in the last period included them. The practice of placing them in the back of the book seems to have become standard, as no text since 1899 has listed them with the individual problems.

Tests and drill materials in arithmetics have also been a recent addition to textbooks. Most of the early texts contained nothing similar to what is today considered test material.

Since the term "test questions" was first found in Milne's book (1877) and "drills" in the Walsh series (1879), these texts should, perhaps, be the first to be listed as containing test and drill material. However, true test material, as it is known today, was not found in

the texts until about 1920. Since that time every arithmetic has included tests of one form or another.

One other type of content, very minor so far as the amount of space devoted to it is concerned, but one which should perhaps be mentioned because it was found in almost every text, was the explanation of symbols. (See Figure 5.)



(13:6)

EXPLANATION OF THE SIGNS.

× Sign of multiplication.

+ Sign of addition.

÷ or) (Sign of division.

= Sign of equality.

The sign of *addition* at the end of an answer, signifies that it is a fraction *too small*.

— This sign signifies, that *that* answer is a trifle *too large*.

: :: : The sign of Proportion, thus, 4 : 8 :: 12 : 24, that is, as 4 is to 8, so is 12 to 24.

⁝ A continued geometrical proportion, or geometrical progression: as 16.8.4.2.1; *i. e.* 16 is to 8, as 8 to 4, as 4 to 2, as 2 to 1.

: or ·.· is an arithmetical equal proportion.

√ Radix, root, or side of a square.

ß signifies semi, or half.

√ signifies that the cube root is to be extracted.

(full size)

Fig. 5. Form in which mathematical symbols were explained in early textbooks

Cocker's Arithmetick, which was written before symbols had been generally accepted, did not use them at all. The only other texts to omit them were Colburn's and two other primary arithmetics published before 1875. Since all three of these texts were designed for small children, it is probable that the authors felt it wise not to add symbolism to an already rather difficult subject. In the last 50 years there have been three or four arithmetics which, while they used symbols, offered no explanation of them. Other than these, all texts have included this type of content.

In early arithmetics symbols were listed and explained on a separate page, usually in the front of the text, a plan followed even as late as 1878. The space given to the subject was never more than a page and often was much less than that. DeMorgan (1832) was the first to include the explanation of symbols with the process when it was introduced. As in the case of answers, the space allotment could not well be measured when each symbol was taken up separately, but it is assumed it was about the same as when they were grouped.

Non-Mathematical Content

In every text there is a certain amount of space taken up by content which is a vital part of the book itself but which does not add to the actual subject matter of the text. Chief among the types of non-mathematical content are the title page, the preface, the table of contents, the index, the appendix, and, in the case of the older arithmetics analyzed, the recommendations.

Title page.—In the books studied there was little variation in the data given on the title page, although the lengthy subtitles of the older texts added considerably to the amount of printed matter on the page. The title, author (sometimes including information about his position and other writings), and publisher were always included.

Preface.—Prefaces, though found in almost every arithmetic, were often very unlike each other. Some authors used them to state the purposes and values of the texts; others discussed the material included, pointing out especially the new phases of the subject introduced; and still others spent most of their time enumerating the faults of other arithmetics and singing the praises of their own. Prefaces varied from only a paragraph or two to several pages in length, the more recent texts usually having the longer prefaces because they included so much more discussion of content and method than did the early ones.

Table of contents.—Seldom was an arithmetic found without a table of contents, although four of the books (Daboll's text, in 1799; French's advanced book, in 1875; Southworth's arithmetic, in 1893-95; and Southworth-Stone's in 1900) used what might more correctly be termed indexes in the front of the book, as they contained alphabetized lists. The only books not having a table of contents were four very elementary books, all published before 1877.

As a rule, the later books had the longer tables of contents, although this was not uniformly true until the last 15 years of the study. In some books the table of contents included only the chapter headings, in others it was detailed, sometimes taking several pages, but the average length during the 150 years was about three pages.

The wording of chapter heads and names of other divisions in the books has changed amazingly through the years, until today it is hard to recognize in the titles the fact that much of the content is approximately the same as in the early texts, presented from a different point of view. In the first texts, very technical terms were used, many of them out of date today, such as "alligation medial," "conjoined proportion," "cross multiplication or duodecimals," "extraction of the sursolids by approximation," and "supplement to contraction in multiplication." Sometimes when more than one topic was found on a page, the page number was given as "*do.*" or "*ibid.*"—a plan which sometimes made it difficult to use the table of contents.

Now and then section headings were very long and all-inclusive.

The first evidence of an attempt at simplification of titles came in S. Pike's book in 1811. Few of his headings were more than two words long—the impression given by his table of contents being that his text was clear, direct, and simple. As headings were shortened they also were worded more simply. "Numeration and notation" became "reading and writing of numbers," "involution and evolution" became "powers and roots," and so forth. Other authors saw the wisdom of this and adopted the idea, though involved headings were found in some texts late in the nineteenth century.

Concise, businesslike headings enjoyed a long period of popularity —material was arranged differently in books, aims were changed, emphasis was shifted, but these trite headings remained in vogue through it all until well into the twentieth century. Early in the 1900's authors began to expand their headings somewhat—instead of just "bills," for example, they said "computing and adding bills." The participial form of expression was adopted very quickly, for it was found valuable in giving the idea of "learning by doing." It was used extensively in the later years covered by this study.

In 1915 the *Everyday Arithmetic* series came out with the first clear evidence of an attempt to socialize arithmetic texts. Headings such as "buying toys" and "weighing groceries" were used in the primary text, but names of parts in the upper-grade books, though somewhat socialized, did not stray so far from the traditional form. This style was not used again until 1928, when the *Triangle Arithmetics* adopted it. Nearly ten years later the *Child-Life Arithmetics* went still further, giving interest-catching names to certain divisions, and carrying the system to some extent even to the eighth grade level. The two most recent series in this study have used combinations of the forms of headings found in all the later texts.

Thus the style of division headings had developed from an involved, technical form to a clear, concise, but very impersonal wording, and finally to a living, personal, and socially interesting idea.

Trends in the organization of the books can also be seen at a glance when the tables of contents are compared. Many of the early authors followed Cocker's example of taking up each new topic in a new chapter, with no attempt at grouping or subordinating topics, but few gave the designation of "Chapter" to their divisions—usually they merely named the sections according to their content. Sometimes the book was divided into "parts," each including several subdivisions, and now and then these subdivisions included sections or numbered articles. DeMorgan's text (1832) was so highly organized along such a plan that the table of contents looked very much like a legal document of some sort, for it was divided first into books, then into sections, and then into many numbered "articles." This idea of including articles was new at the time, but was adopted by many later authors, none of whom, however, saw fit to list such a detailed breakdown of the subject matter in the table of contents.

By the middle of the nineteenth century the majority of the books were using the form of large divisions, each including subtopics, but few of these used the designation "chapter" until after 1900. In most of the newer texts, each volume of the series was broken down into chapters and a table of contents covering that volume was included in the front of each book. On the whole, the tables of contents indicate that more thought has been given in recent years to the organization of the textbook.

Index.—Until French's *Common School Arithmetic*, in 1866, none of the books studied contained an index, unless the table of contents found in the back of Hassler's *Elements of Arithmetic*, in 1826, could qualify as such. As has been said, four texts were found with indexes in the front, but the use of an alphabetized list of items included in the book was rare until after 1895. Since 1900 it has become a standard part of an arithmetic text and is included in at least the advanced books of every series in this study except one published after that date.

Appendix.—An appendix was found in about one fourth of the books, scattered throughout the 150 years covered by this study. No trends at all were evident except, perhaps, the tendency for appendices to be dropped from the more recent arithmetics, as only one series since 1906 included one, and in this text, the *Progressive Arithmetics*, it was entitled "Supplement." The appendices varied from four pages to 49 pages in length and dealt with a variety of subjects. Definitions and measures made up the content more often than did any other subject. Sometimes, however, material not closely related to the required work of the text was included, such as algebra, geometry, bookkeeping forms, or short cuts. Such supplementary material was not often found in recent elementary school texts. When given, however, it was presented as a part of the required work in the texts designed for the grade or grades in which the topic was placed.

Recommendations.—The only other type of non-mathematical content found in texts was included only in the older arithmetics. This was a sort of testimonial kind of advertising usually in the form of

letters written to the author by persons in a position to pass judgment on the worth of the text. These usually were found either immediately preceding or following the preface, but were now and then on the back of the book or on the outside of the back cover.

Cocker included only the names of persons who had endorsed his book—names of prominent persons, among them Noah Webster. Authors who followed him, however, printed copies of some of their most cherished letters. Such comment as the following was found:

> Providence, State of Rhodeisland, 1785.
>
> WHOEVER may have the perusal of this treatise on Arithmetic [Nicholas Pike's *A New and Complete System of Arithmetic*] may naturally conclude I might have spared myself the trouble of giving it this recommendation, as the work will speak more for itself than the most elaborate recommendation from my pen can speak for it: But as I have always been much delighted with the contemplation of mathematical subjects, and at the same time fully sensible of the utility of a work of this nature, was willing to render every assistance in my power to bring it to the public view: And should the student read it with the same pleasure with which I perused the sheets before they went to the Press, am persuaded he will not fail of reaping that benefit from it which he may expect, or wish for, to satisfy his curiosity in a subject of this nature. The author, in treating on numbers, has done it with so much perspicuity and singular address, that I am convinced the study thereof will become more a pleasure than a talk.
>
> The arrangement of the work, and the method by which he leads the *tyro* into the first principles of numbers, are novelties I have not met with in any book I have seen. Wingate, Hatton, Ward, Hill, and many other Authors, whose names might be adduced, if necessary, have claimed a considerable share of merit; but when brought into a comparative point of view with this treatise, they are inadequate and defective. This volume contains, besides what is useful and necessary in the common affairs of life, a great fund for amusement and entertainment. The Mechanic will find in it much more than he may have occasion for; The Lawyer, Merchant and Mathematician will find an ample field for the exercise of their genius; and I am well assured it may be read to great advantage by students of every class, from the lowest School to the University. More than this need not be said by me, and to have said less, would be keeping back a tribute justly due to the merit of this Work.
>
> BENJAMIN WEST (3:3)

> Having been made acquainted with Mr. Colburn's treatise on Arithmetic, and having attended an examination of his scholars, who had been taught according to this system, I am well satisfied that it is the most easy, simple, and natural way of introducing young persons to the first principles in the science of numbers. The method here proposed is the fruit of much study and reflection. The author has had considerable experience as a teacher, added to a strong interest in the subject, and a thorough knowledge not only of this but of many of the higher branches of mathematics. This little work is therefore earnestly recommended to the notice of those who are employed in this branch of early instruction, with the belief that it only requires a fair trial in order to be fully approved and adopted.
>
> J. FARRAR (9:ii)

Advertising in the form of relating the popularity of the text as judged by number of editions published, number of copies sold, or number of cities authorizing the text was not found often in the prefaces of older books and not at all in recent texts. With large publishing companies producing the books and the policy of state-wide adoptions generally accepted, book agents have for many years contacted only those who were influential in these adoptions, so that it has no longer been necessary to present recommendations to the individual teacher.

The non-mathematical content of the books has always been more or less fixed, and therefore the arithmetics have not varied greatly in this respect. There has been an attempt, however, to make prefaces more helpful to teachers, to make the tables of contents more attractive in appearance and more interesting in the way headings are worded, and to include indexes to make the texts more usable.

Variety of Content in Each Text

The number of textbooks containing each type of content is of special importance not only in showing the extent to which authors have agreed through the years as to what material should be a part of a good arithmetic, but also in indicating trends in this opinion. This information also shows which topics have been dropped when outmoded or no longer considered a part of a course in elementary arithmetic, and which ones have been added as the need for them has arisen.

In Figure 6 these trends can be traced clearly so far as certain content is concerned. Since not nearly all the content of the texts could be classified and included in the chart, it must be remembered that this is not a complete picture of the arithmetics. However, all types of content given considerable space in the texts are included, as well as a number of the topics which have been popular in their day but which have been found only in early or late texts. These have all been discussed in the comment on the various types of material included in texts.

From the information given on this chart another interesting and fairly accurate comparison of content material can be made, this time from the point of view of the variety of content included in each text. On the whole, there seems to be greater variety in the more recent books.

Cocker's Arithmetick (1677) included only 16 of the 26 different phases of arithmetic pictured on the chart; only 5 of the 16 kinds of business transactions; and none of the daily-life activities. Pike's text (1788) and Gough's (1801) contained many more than this, but Colburn's (1821), which was designed especially for young children, contained very few. In the field of business practices, for example, Colburn included only interest and profit and loss. DeMorgan's text (1832) was another which contained only a few of the types of content, but Thompson's (1825) presented most of those found in other texts of his day.

Texts of the last 20 years have contained most of the kinds of material found on the chart except for the types that have not been

Fig. 6. Graph of certain kinds of arithmetic content, showing in which textbooks each is included

included in any texts since the turn of the century. On the other hand, no text before 1862 included any of the topics considered as daily-life activities and Dean's text was the only one before 1898 to include more than two of the ten listed on the chart. Only three texts, all published since 1930, included all topics in this classification.

From space counts given in the discussion of content and from evidence as pictured in Figure 6, it seems that recent textbook writers are attempting to include in their arithmetics a greater variety of usable material and to omit those principles and processes that are no longer needed by the average person. The most startling change in content has of course been in the field of daily-life activities, a phase of arithmetic which has been introduced into textbooks as a part of the movement toward socializing arithmetic and making it understandable and usable to the school child in his everyday experiences.

Chapter V
Method

At the time the first textbooks in this study were published, content was considered a far more important factor in arithmetic texts than was method. Many phases of the content had already been established as desirable and necessary and were included as a part of almost every arithmetic published. Scientific studies of the results of teaching by various methods had not yet been made, and the way in which the subject was taught was a more or less individual responsibility. Teachers who taught from these texts used whatever techniques they could devise, or exchanged ideas with one another. Usually these fell into the pattern of methods with which the teacher was acquainted as a school child. Seldom did an author stray from the traditional deductive method of presenting material, and little progress took place.

The first major change in methods in teaching arithmetic during the 150 years covered by this study came about with the publication of Colburn's *Intellectual Arithmetic*, in 1821. Here was the first American textbook in arithmetic to consider the pupil more important than the subject. The desirability of many of the characteristics of his text was recognized almost immediately, and his influence has been felt throughout the years. In fact, as recently as 1937, the authors of the *Child-Life Arithmetics* felt this influence so keenly that they stated that "modern arithmetic in America is the lengthened shadow of Warren Colburn. It was he who in 1821 broke the monopolistic fetters of adult standards in this subject" (57b:iii).

With the entrance of the Pestalozzian influence in the teaching of arithmetic came the beginning of a shift of emphasis from subject matter to the child himself, a change which has been found in increasing degree in textbooks up to the present time. For many years any adjustments that were made in texts for the sake of the child, such as clarifying and simplifying the rules and vocabulary, were based solely on the judgment of the author, but by the middle of the century scientific studies were being made of the learning process, of child development, of psychological principles, and, later, of individual differences. All these had their effect on the textbooks in the field.

Great strides have been made in textbook methods since Colburn's day and, as a consequence, important changes have come about in the content included in arithmetics and the way in which it is presented. There is still room for improvement, as every arithmetic teacher agrees, but the developments of the last century in this respect offer a very hopeful picture of the possibilities for the future in discovering means of instructing children in such a way as to make them enjoy the subject, and in providing them with a mathematical background sufficient to carry on their everyday affairs satisfactorily.

Role of the Teacher

As has been said, early arithmetics were designed especially for the businessman or the artisan who, when he found it necessary to know more about the mathematics involved in his job, had only the textbook to guide him. The teacher was not an important factor at first, and many textbook authors did not consider him in their presentation of the subject. Now and then the student sought the help of some person better versed in arithmetic who could clarify the material for him, but such "teachers" were individuals who had studied the subject because of a personal interest or need for it rather than those especially trained to teach arithmetic. They used no scientific method of teaching—they simply gave help where it was needed.

About the first 30 years covered by this study were a part of what was known as the "ciphering book" period, when the schoolmaster's work consisted solely in "setting sums" for the pupils to copy and in checking on the correctness of their work. Often the instructors were not qualified to teach, for many of them had memorized their arithmetic without fully comprehending it. They dictated or copied rules and problems from the "sum books" which they themselves had made as students and passed them on to their own pupils for practice. However, if the teachers memorized their rules and definitions carefully and were accurate and neat in copying their work, they could do much toward helping others get their sums correct, even if they did not understand the principles back of them.

As the idea of public education grew in popularity and school attendance increased, the task of setting sums became too time-consuming for the schoolmaster, and arithmetic texts for pupils gained in favor. For many years, however, the same method of teaching was used as had been popular earlier. This method would today perhaps be termed a textbook method, with the teacher offering nothing to supplement the work.

Until early in the nineteenth century arithmetic had been looked down upon. It was a part of the routine necessary in carrying on business and was handled by employees rather than by owners of the larger industries, and therefore the aristocracy felt little interest in learning the subject. But after the beginning of the industrial age, with its tremendous increase in manufacturing and travel, the need for at least a rudimentary knowledge of arithmetic on the part of everyone was recognized.

About this time compulsory education was becoming popular and was spreading from one state to another, although the school laws were not as yet very effective, and increasingly large numbers of children were being sent to school instead of educated in the home. Arithmetic was a required subject in most of these schools, and a definite need for teachers in the field made itself felt.

Colburn (1821) was one of the first to advocate class instruction. In his text he recommended a form of the proctor system in order that teachers might handle more pupils in their classes. He suggested that the teacher "let the elder pupils hear the younger" as "a useful exercise for them, and an assistance to the instructer" (9:143). This plan eased

the problem of the shortage of teachers, but the rate of increase in pupil population far exceeded that of teaching personnel.

On the whole, early textbook writers had a better knowledge of method than did teachers, for they at least had ideas as to techniques for presenting the content of their books. As a consequence, they began to include notes to teachers and pupils explaining ways in which certain material might be used. There was full recognition on the part of these authors that the teachers were not properly equipped for their work, and the notes were often in the form of encouragement. Colburn, for example, in introducing fractions, stated that:

> Instructers, who may never have attended to fractions, need not be afraid to undertake to teach this book. The author flatters himself that the principles are so illustrated, and the processes are made so simple, that any one, who shall undertake to teach it, will find himself familiar with fractions before he is aware of it, although he knew nothing of them before; and that every one will acquire a facility in solving questions which he never before possessed. (9:141)

Now and then, throughout his book, Colburn assured the teacher that the examples were easy and that, when presented right, "all difficulty will vanish." He included a key at the back of his text for the sake of the instructor who did not understand the work, and in some cases went into much detail in suggesting certain techniques. Here again he gave all the aid he could to the untrained teacher.

In 1827, Adams encouraged teachers to get more training in subject matter before attempting to teach:

> Then let teachers make themselves, in the first place, thoroughly acquainted with arithmetic. The idea that they can "study and keep ahead of their classes," is an absurd one. They must have surveyed the whole field in order to conduct inquirers over any part, or there will be liability to ruinous misdirection. (17:1*)

By the middle of the century the need for better trained teachers was being felt very keenly. Authors were giving as much assistance as they could in the form of suggestions and devices to be used and questions to be asked, but these must have been very inadequate for a teacher who was not resourceful. For instance, French included such notes to teachers as: "Show wherein these General Principles [of finding the least common multiple] are similar to the General Principles of Division" (29:vii). A teacher who had received no further training in arithmetic than that gained in elementary school and possibly a few weeks at an academy would not get a great deal of help from such a suggestion. Other authors evidently felt the greatest help they could give the teachers was in providing them with a plentiful supply of questions. In some texts this was done by including a good many written problems following each new topic discussed and then giving several pages of "promiscuous" or miscellaneous questions at the end of the text. Some authors handled it differently, listing review questions for the use of teachers and pupils in footnotes throughout the book.

Even as late as 1866 the lack of teacher preparation was evidently of great concern to textbook writers. French, for example, stated in

his elementary text that "when either pictures or examples contain objects or terms with which a child is unacquainted, explain them to him, consulting a dictionary whenever you do not understand the object to be explained, or the term to be defined" (29a:108). Student teachers today are taught such things in methods courses, and are not supposed to be in need of elementary instruction by the time they are ready to take charge of a class. Certainly no mention would be made in a modern textbook of the fact that the teacher might not understand the work presented for the pupil's education.

While early texts were written for the student, most of the arithmetics between 1820 and 1880 seem to have been designed for the use of the teacher rather than of the pupil, even though in the more recent texts of the day an attempt was being made to keep the content on the level of the pupil. In the latter half of the nineteenth century emphasis began shifting from the teacher back to the pupil. White (1870) made it clear in his preface that his text was designed as a drill book for pupils, not as a manual for teachers.

The instructor then began to play a less important part in the pupil's relationship to his textbook. By 1913 the teacher was told that "the pupil naturally prefers to do the work himself at his desk rather than to be helped in the recitation by the teacher" (50a:iii). Authors were beginning to take advantage of the findings of scientific studies on child psychology and the learning process, and were presenting their material in a form both interesting and understandable to the pupil. With textbooks organized on this basis, the role of the teacher in determining teaching techniques became still less important. Today he acts almost entirely as guide or helper, and the pupil is again learning much of his arithmetic direct from the textbook, though it is a very different textbook from the one used 150 years ago.

Methods of Presenting Material

Although a great many of the devices and techniques used in teaching the various phases of arithmetic have recurred over and over again, these have all been influenced by certain major movements in education. In the teaching of arithmetic there have been very definite "styles" at different times, and the majority of textbooks of the day have followed these in their organization and presentation of material. (See Figure 7.)

Deductive method.—During the first period covered by this study all textbooks in arithmetic used the deductive method. According to this method, each new topic was begun with a definition and rule. This was followed by an example, with the details of its solution presented. Sometimes this was accompanied by a full explanation, sometimes not. Then followed practice, or problems to be solved by the use of the rule given. Some authors referred to these as "application" because they were problems showing how the rule could be applied to business practice. Pupils were not required to make any decisions for themselves as to the processes involved in working a problem, for the type of exercise provided always fell under the rule being taught.

When the deductive method was in vogue, topics were presented in what was believed to be the order of difficulty. Each topic presented was supposed to involve slightly more complicated processes than the preceding one. In a few cases the topics were grouped under four or five main headings, such as whole numbers, fractions, decimals, etc., but more often no arrangement of material other than order of difficulty was evident. To take care of all the possibilities of irregularities in problems encountered, separate rules were given for each variation of a general rule, and, for each of these, rules of proof were usually included. Since the rules were to be learned word for word, the pupil found that much of his study of arithmetic was memory work.

With the rising need for teaching arithmetic to children of school age, the deductive method, which lent itself especially to adult teaching, became severely criticized. Colburn, in the preface to his *Intellectual Arithmetic,* expressed his reasons for objecting to the plan:

> The usual mode of proceeding is as follows. The pupil learns a rule, which, to the man that made it, was a general principle; but with respect to *him*, and oftentimes to the instructer himself, it is so far from it, that it hardly deserves to be called even a mechanical principle. He performs the examples, and makes the answers agree with those in the book, and so presumes they are right. He is soon able to do this with considerable facility, and is then supposed to be master of the rule. He is next to apply his rule to practical examples, but if he did not find the examples under the rule, he would never so much as mistrust they belonged to it. But finding them there, he applies his rule to them, and obtains the answers, which are in the book, and this satisfies him that they are right. In this manner he proceeds from rule to rule through the book.
>
> When an example is proposed to him, which is not in the book, his sagacity is exercised, not in discovering the operations necessary to solve it; but in comparing it with the examples which he has performed before, and endeavoring to discover some analogy between it and them, either in the sound, or in something else. If he is fortunate enough to discover any such analogy, he finds what rule to apply, and if he has not been deceived in tracing the analogy, he will probably solve the question. His knowledge of the principles of his rule is so imperfect, that he would never discover to which of them the example belongs, if he did not trace it by some analogy, to the examples which he had found under it. (9:viii-ix)

As Colburn said, ability to understand the reasons for the processes used in the solving of different kinds of problems was not considered an end in the teaching of arithmetic. No attempt was made to help the child "think through" a problem or determine the best means for solving it. This lack of training had been felt very keenly by some educators, who welcomed Colburn's method of meeting the problem and followed his lead in breaking away from a purely deductive presentation.

Inductive method.—Colburn's text revolutionized ideas concerning method in teaching arithmetic, for his presentation was purely inductive —at the other extreme from the deductive plan used up to his time. He built up a need and an understanding on the part of the pupil so

gradually and so naturally before presenting rules or definitions that, by the time the pupil was ready to generalize, he could state the rule himself.

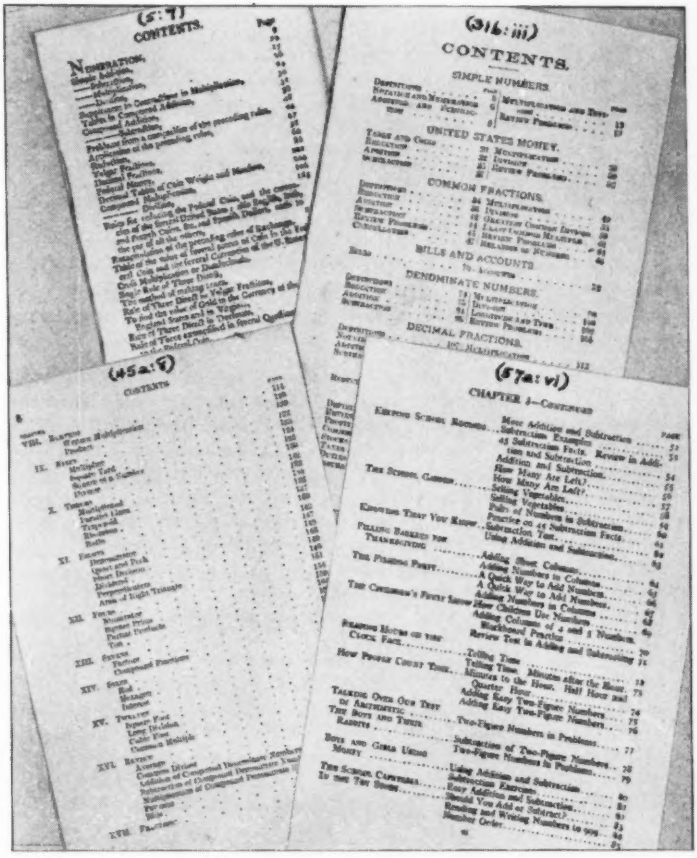
Colburn was strongly influenced by Pestalozzi. In his preface he stated that "in forming and arranging the several combinations, the author has received considerable assistance from the system of Pestalozzi" (9:ix). The principal idea was to teach arithmetic to the young child in such a way as to develop a thorough understanding, having him "first perform the examples in his own way, and then be made to observe and tell how he did them, and why he did them so" (9:1*). Beginners were introduced to a very simple kind of arithmetic. In the first 50 pages of his text no symbols were used and numbers were written out in words instead of in figures in order to keep the difficulty of the problems on a more elementary level. Problems for beginners dealt with concrete objects—the first question in the book being "How many thumbs have you on your right hand? how many on your left? how many on both together?" (9:2). After an understanding of number was built up in this way, what might be called drill problems were given, such as "Two and one are how many?"

He took up topics in much the same order as did earlier writers, but did not include any work more advanced than fractions. In later textbooks written on the inductive plan the order in which topics were included was sometimes varied, and additional topics were discussed, extending into more advanced work.

Many who followed Colburn used his method in every detail, and almost all later arithmetics were influenced to some extent by his work. Some of the educators who were wholeheartedly in favor of the new system of teaching arithmetic carried it into a new phase of method, organizing the subject matter in a logical order, and presenting it, by the inductive method, a topic at a time.

Topical method.—Chase (1848) was one of the first to show a decided tendency toward a topical scheme of presentation. His content was arranged topically, but in introducing each new topic he combined the deductive and inductive methods. Others before his time had divided their content into main divisions with subtopics, but Chase broke up the subject matter into a larger number of units. His text contained 15 chapters, each carefully organized according to a definite plan. The first chapter was on numeration, divided into five subdivisions; the next four were on addition, subtraction, multiplication, and division, respectively, each beginning with a subdivision entitled "Definitions, and Signs"; then came other chapters all presented in much the same form.

No other texts until Felter's *Analysis of Written Arithmetic* (1862) and Dean's *Intermediate Arithmetic* (1865) used an extreme form of the topical plan, although many writers recognized its value and followed it at certain grade levels. The principal advantage of this method was that it gave what was considered a thorough groundwork in one phase of arithmetic before taking up the next. While the plan was frowned upon by a few later authors, it is still used for the ad-



(approximately one third size)

Fig. 7. Tables of contents of texts of various periods, showing the arrangement and organization of topics

vanced grades in some texts, for it lends itself well to the program of letting each child progress at his own speed.

Grube method.—By 1895 a few texts showed evidence of the use of the Grube method, a concentric plan in which the study of one number was exhausted before the next was taken up. The elementary texts in the Indiana Schoolbook series (1897-98) and the Hornbrook series (1898-1900), both of which were written by Hornbrook, went to the extreme in this respect, as each of 11 of their 17 chapters was concerned with one specific number. For example, Chapter II dealt

with tens, Chapter III with twos, and so forth. Each chapter was given approximately the same amount of space, with the space increasing slightly as more difficult numbers were taken up, and the discussion of all the fundamental processes in relation to each number was completed within that chapter before the next number was discussed. The order in which the numbers were taken up was supposedly that which would be most economical of the pupil's time. Tens came first, then twos, threes, fives, elevens, nines, eights, fours, sevens, sixes, and twelves. The order was determined somewhat by the relationship of numbers to each other and somewhat by their use in common measurement. At the end was a chapter of general review and one which introduced fractions. Only work through the fourth grade was presented in this way, the advanced book of the series being arranged topically. However, few texts were found in which the Grube method was very evident.

Spiral method.—The topical method had not proved entirely satisfactory in that students were at times rushed into rather difficult work in completing the study of a certain topic before being introduced to the simpler elements of another topic, and for a number of years toward the end of the nineteenth century there was evidence of an attempt to get away from a purely topical form of presentation. The spiral method seemed at the time to be the answer to this problem, a method in which all simpler phases of a topic were taken up first, with more complicated material introduced each time the topic recurred. In some texts topics reappeared many times, in others they were not included so often and were dealt with more fully each time they were discussed.

The McLellan-Ames series was one of the first to use a definitely spiral method. In the elementary book the first section dealt with a review of first grade work. This included nothing more than learning to count and to perform the four fundamentals, using small numbers. In the second section a more extensive discussion of addition, subtraction, denominate numbers, quantity, units of measure, and number was given. The most elementary phases of all of these had been included in the first section. This means of constant review and extension of topics was carried on throughout the elementary text; the advanced text, however, was arranged according to topics. Even as recently as the *Daily-Life Arithmetics* (1938) this method has been used especially in presenting number facts—taking one number at a time—and in giving the review of previous work at the beginning of each volume before going on to more advanced work.

The spiral method had many advantages, one of the most important of which was that the child was provided with fundamental knowledge of the elementary phases of different processes in arithmetic while he was in the early months of his schooling. New content could then be graded according to difficulty and presented at the school level where it could best be understood. In the upper grades, where the various topics introduced required more nearly the same level of ability and knowledge of arithmetic, the topical plan was still used in most texts. However, on certain levels the spiral method was found in a modified form in almost all the later arithmetics in this study.

By 1904 authors were beginning to question the value of the extreme spiral method and to seek a "happy medium" between the topical and the spiral. Smith, in his *Primary Arithmetic* published at that time, made the statement that:

> The purely "topical method," the attempt to exhaust a subject like common fractions in a single chapter, is now obsolete in our leading schools, while the extreme "spiral method" is scrappy, uninteresting, and lacking in the continuity so essential to thoroughness. Between these two comes the best type of our modern courses of study, somewhat spiral in arrangement, in that most subjects extend over several terms, but admitting of a topical arrangement within any one term, thus securing thoroughness and maintaining an interest. (48a:iii)

Activity method.—A combination of former methods was based on a more thorough understanding of psychology, and developed into what might be called the "activity" method. Child activities and interests came to be used as a background for presenting content on the theory that students learned more readily if they were interested and that they were more likely to retain what they had learned if it had involved physical activity as well as mental manipulation.

The Hornbrook series (1898-1900) was one of the first to make any extensive use of the activity method. This text contained many suggestions to teachers of devices for making the lesson more alive. Instructors were advised to "let the children suggest different rectangular solids to be placed in the room, and find the cu. ft. of air displaced by them" (45a:188), to find the differences between the weights of children in the class, to beat time (one to a second) to get an idea of the length of a minute, etc. In this way the child was being given an active part in the lesson and was shown that there really was a need for arithmetic in everyday life.

The imaginary situation was the one most often used at first, although a few books contained suggestions like that found in the Woodward series, that the children be allowed to pour water into measures of various sizes in learning their table of liquid measure. The authors called attention to the fact that this was learning by doing and that an experience was more easily recalled than was a statement to be memorized.

Such activities were used to connect arithmetic with the life of the child, to provide him with a natural background for new information and processes. This led to the grouping of problems around a central theme, using the same setting for all, such as gardening, cooking, or working in a store.

Sometimes geographical, historical, or scientific information provided the background for several pages of problems. Some books carried this method into a "storybook" style, grouping their problems around the experiences of members of a certain family. The activity element in such a presentation of material evolves from the fact that the child becomes so surrounded by the "story" that he takes an active interest in the solution of the problems for the sake of filling in the gaps in the central theme of the unit.

Except for the attempt to create an interest and to teach the child through activity wherever possible, the activity method could not be said to have any set form of presentation. If the inductive method seemed most suitable, it was used; if the spiral method was most useful, it was adopted. All methods that had proved satisfactory in the past shared in creating an activity method—no longer was the presentation of content made to follow any set form.

There is a tendency on the part of people in every age to think the method they use is better than any method formerly popular. Strengths and weaknesses are always more clearly seen at a distance. As each new method was adopted in the teaching of arithmetic, authors sang its praises and pointed out the weaknesses of the old. Recent textbook writers have been strongly in favor of the interest and activity elements in the new method, and have called attention to them in their prefaces. The *School Arithmetics* (1919) stated their attitude clearly, a point of view held by educators ever since.

> There are various extreme positions from which the preparation of such a book may be approached, but, as is always the case, there is also a safe mean which is certain to produce satisfactory results. One extreme makes the arithmetic operation the sole purpose, neglecting to a large extent those applications that appeal to children. Another extreme attempts to make the application the sole purpose, neglecting that steady advance in power to compute that gives to pupils the consciousness of definite progress. A third extreme seeks to make all arithmetic a matter of play without serious purpose, and still another extreme attempts to frame a textbook that is merely a collection of methods for teachers, a book that is weak in drill and weak in serious applications. No one of these extremes has ever given or, for psychological reasons, can ever give satisfactory results.
>
> The safe mean proceeds on the supposition that the pupil should be led to his arithmetic through paths which are interesting; that he should see that he is studying a subject which is usable in school, in his play, in his home, and in all other phases of his daily life; and that, so far as possible, the applications should be real to the pupil, particularly in those grades in which his tastes are being formed and in which his outlook on life is very limited. (52a:iii)

This "safe mean" continues to be the basis used in writing textbooks. Changes have come about in techniques and general style of texts, but the human interest element and the idea of presenting material so that the child can "learn by doing" are still uppermost.

Important Teaching Theories and Techniques Prominent in Texts During the Nineteenth Century

Interwoven with the major methods of presentation already discussed were many other important teaching techniques or theories which were strongly in the limelight at one time or another through the total period covered by this study. A few of the larger movements were evident in the textbooks analyzed.

Object method.—Until Colburn's time little use was made of the object method in arithmetic textbooks, but it must be remembered that

early texts had been written for adults, and adults knew how to count and to make simple additions before beginning the study of a textbook. It is in the beginning stages of learning in arithmetic that the object method has proved most valuable.

Before 1821 the study of numeration in texts had begun with a rather philosophical discussion of the meaning of number, but Colburn recommended a more objective method, with examples being "performed in the mind, or by means of sensible objects, such as beans, nuts, &c., or by means of the plate at the end of the book" (9:1*). He used Pestalozzi's plates for teaching numerical relationships. Plate I contained 10 rows of rectangles, 10 in a row, with "1" marked in each rectangle in the first row, "2" in each in the second row, and so forth. Plate II was made up of the same 100 rectangles, with each in the first row whole, each in the second row divided in half by a vertical line, each in the third row divided into thirds in the same manner, and so forth. Plate III, the last of the plates, was an extension of Plate II, the rectangles being divided by dotted horizontal lines in the same way as they were divided by vertical lines in Plate II. Thus Colburn carried his object teaching beyond beginning number.

Through every period taken up in this study a number of authors were found favoring the object method. DeMorgan (1832) said, in a note, "any little computations which occur in the rest of this section may be made on the fingers, or with counters" (15:12). French (1866) brought in also the idea of teaching more advanced work through the handling of objects when he said:

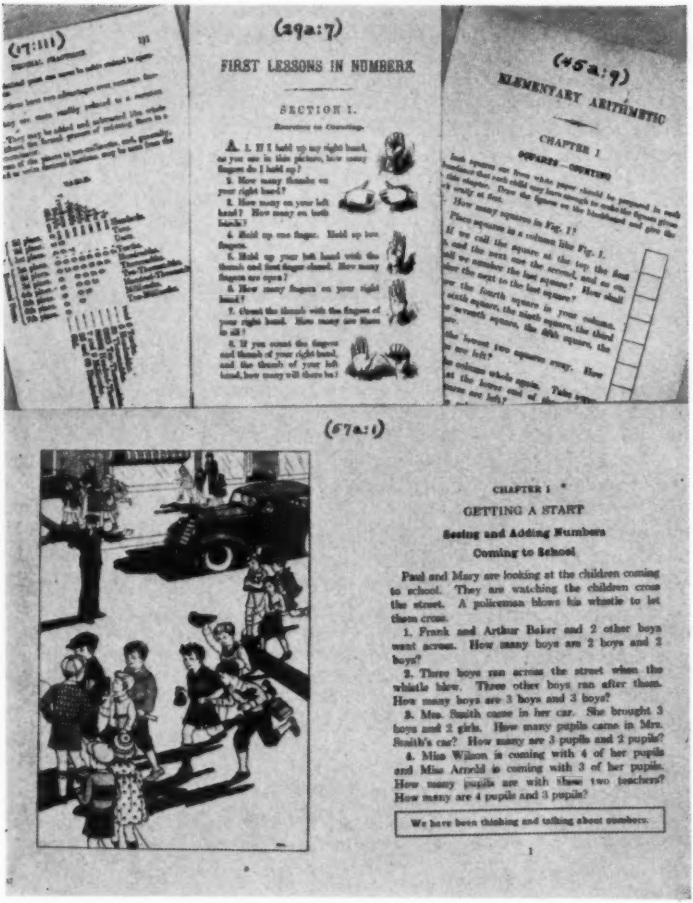
> Before assigning a table as a lesson to be learned, explain its uses, and the several denominations, using visible objects in every case where it is practicable. Also require each pupil to handle the weights and measures, and to use them in weighing and measuring various kinds of objects. (29a:118)

In his preface, French suggested that:

> Every primary school-room should be supplied with a variety of kinds of objects . . . to be used by the pupils in learning arithmetical combinations. For this purpose, walnuts, horse-chestnuts, or buckeyes, pebbles, seashells, large flat beans, blocks of wood, etc., may be used. (29a:2)

When illustrations became popular, all kinds of things were pictured for the pupil to count—apples, stars, wigwams, sticks, pennies, etc. The Johnson series (1900) added to the idea of using objects the belief that they should be presented in such a way as to stimulate and develop the ability to discriminate and see relationships. A very few years later the Smith series showed definite evidence of trying to carry out such a plan by teaching fractions, surfaces, and cubic measure through having the children actually fold and cut paper by measurement. Many texts before and after this time had used the idea of dividing paper into sizes showing relationships, but it had taken the form of a discussion rather than an activity. (See Figure 8.)

Apparently some educators, even earlier than this, had begun to feel that too much stress was being placed on the use of objects, for a dissenting note was found here and there. In the Cook-Cropsey


(29a:7)
FIRST LESSONS IN NUMBERS
SECTION I.
(45a:9)
ELEMENTARY ARITHMETIC
CHAPTER 1
(67a:1)
CHAPTER 1
GETTING A START
Seeing and Adding Numbers
Coming to School
Paul and Mary are looking at the children coming to school. They are watching the children cross the street. A policeman blows his whistle to let them cross.
1. Frank and Arthur Baker and 2 other boys went across. How many boys are 2 boys and 2 boys?
2. Three boys ran across the street when the whistle blew. Three other boys ran after them. How many boys are 3 boys and 3 boys?
3. Mrs. Smith came in her car. She brought 3 boys and 2 girls. How many pupils came in Mrs. Smith's car? How many are 3 pupils and 2 pupils?
4. Miss Wilson is coming with 4 of her pupils and Miss Arnold is coming with 3 of her pupils. How many pupils are with these two teachers? How many are 4 pupils and 3 pupils?
We have been thinking and talking about numbers.
1

(three eighths size)

Fig. 8. Various methods of teaching numeration

series (1892-93), for example, a note to the teacher stated that "if necessary, the work must be given with the objects. As soon as possible, however, children should become independent of the use of objects" (49a:3).

In the most recent texts number has been taught through the use of stories—a story of a child's party, a picnic, a circus, and so forth—in which the child was to tell how many children, or hats, or boys, or sandwiches there were. While the teaching in some of the newer books seemed to be slightly more abstract than was teaching in many

of the earlier books, still the pictures helped to give an objective tone to it.

Inclusion of answers.—The advisability of including answers to problems is and has been for many years a moot question. In the early texts included in this study, answers were always given; then, after several years of disagreement among writers, came a period when very few texts were so "old-fashioned" as to include answers; and recently, in the last period considered in this study, the pendulum seemed to be swinging back again.

Cocker's text (1677) included explanations as well as answers, and Colburn's (1821) gave the answers with the questions but included a key in the back of the book in which type questions were explained, probably for the sake of the teacher. Answers almost always followed the questions in these early texts, and were usually designated as "Ans." However, in two or three of the earliest arithmetics, the term "Facit" was used, often with problems involving measure, and now and then a text was found which used the words "Result," "Amount," "Product," "Quotient," or some such designation.

The first text to express in its preface its disapproval of the method of including answers was *The Youth's Arithmetical Guide* (1805), which included the statement:

> The authors of the following compend have long been united in the opinion, that the practice, so generally adopted, of giving questions in arithmetic with the answers annexed, though extremely convenient to teachers, is by no means the most profitable for learners. This opinion is founded on the following considerations:—
>
> 1. That the proper business of arithmetic is, not to find results already known, but to discover those which are unknown.
>
> 2. That having the answer given, serves in some measure, as a guide and operates to the disadvantage of the learner, in every rule; more particularly in the rules of proportion, where by the aid it affords in making the statement, and finding the result, it prevents that careful examination, which is so essential to a clear and profitable understanding of the question.
>
> 3. That pupils accustomed to work such questions only, as have the answers given, seem utterly at a loss when desired to work those to which they are not given.
>
> And that learners, by having answers annexed to the questions, are often enabled (perhaps tempted) to impose upon their teachers, and thus wrong themselves, by exhibiting true results without having fairly obtained them. (7:v)

The authors then went on to state that manuscript questions had been used as a means of avoiding these difficulties, but that they had been found so burdensome and inconvenient that it was decided to offer the schools an arithmetic without answers to its questions. However, printed copies of each of these arithmetics, with the answers included, could be obtained if desired.

In 1848, when authors were disagreeing about the matter, Chase tried to compromise on the subject. Instead of including answers with

the questions, he grouped them on several pages in the back of his *Common-School Arithmetic*. He had evidently given the matter considerable thought, for he stated on the back of the title page to the key:

> The attempt to prevent the use, by scholars, of any work which is designed "for teachers only," nearly always fails, and the author has known of so many instances in which a habit of deceit has been strengthened by such an attempt, that he thinks it best to place the Key in the hands of every pupil. But in order that the learner may find some necessity for self-reliance, the answers to many of the questions have been purposely omitted. It may be well for every teacher to enter all the deficient answers in a blank book, (which can be done with very little labor,) and he can then easily assure himself whether his pupils work understandingly, or are in the habit of *forcing* their results. (18:258)

The first text not to include answers was Davies' *First Lessons in Arithmetic* (1852). A teacher's key would of course not be necessary for such elementary work, but answers were included in his more advanced text. After this, it became rather common practice to include only part of the answers to problems. The only other texts before 1897 for which no answers were given were Felter's (1862) and Mecutchen-Sayre's (1877), neither of which so much as mentioned answers.

The Woodward series (1899) was the last to include answers with the questions, as the practice of putting them in the back of the book had become quite common by that time. Only about ten of the texts published since the beginning of the century have included answers, but the advisability of this practice still seems to be questioned, since the last two books used in this study disagreed on the subject.

Use of analysis.—Adams credited Warren Colburn with having introduced the analytical method of instruction in arithmetic "with ingenuity and success" (17:iii). This method came as a reaction to the memory work of the first period of this study, when examples were given with little or no explanation of the reasons for the various steps in their solution. A great surge of interest in developing within the pupil an understanding of arithmetic swept through the schools. By the time Adams published his text (1827), the popularity of the analytical method of teaching was in full swing, and for the next 30 or 40 years most texts pointed out in their prefaces, and sometimes in their titles or subtitles, that they were written on the analytical plan.

The value of the analytical method was recognized particularly in the fact that it helped lead the pupil to attack a new problem through reasoning rather than through memorization only. Felter, in 1862, expressed this idea in the preface to his text:

> While undue prominence should not be given to this method, it is important that in the analysis of problems the pupil be continually required to point out the more important of these elementary questions, and the formulas involved in them. By this means, he will become self-reliant, unaccustomed to flee for refuge in every difficulty to some RULE; but will depend upon the exercise of his reason for a solution. (24:vi)

In his text he went to the extreme, including each minute step and repeating constantly in order that every statement might be complete within itself. A typical example of the detail he used in his analyses follows:

PRACTICAL EXAMPLES COMBINING ADDITION AND SUBTRACTION AND CONTAINING TWO OR MORE ANALYTICAL STEPS.

A farmer sold a span of horses for $138, a cow for $38, some butter for $13, and some straw for $15; he purchased 10 yards of broadcloth for $40, a stove for $35, some milk-pans for $13; how much money had he left?

MODEL OPERATION

(a.)			(b.)		
$138	received for	horses.	$40	paid for	broadcloth.
38	"	cow.	35	"	stove.
13	"	butter.	13	"	milk-pans.
15	"	straw.			
$204	received for all.		$88	paid for all.	

(c.)

$204	amount	received.
88	"	paid.
$116	amount	left.

ANALYSIS

ANALYTICAL STEP. (a.) Find the amount of money received.

Elementary Question.—If a farmer sold a span of horses for $138, a cow for $38, some butter for $13, and some straw for $15, how much did he receive?

Arithmetical Formula.—If a farmer sold a span of horses for $138, a cow for $38, some butter for $13, and some straw for $15, he must have received for all the sum of these amounts, which is $204.

ANALYTICAL STEP.—(b.) Find the amount of money expended.

Elementary Question.—If a farmer purchased 10 yards of broadcloth for $40, a stove for $35, and some milk-pans for $13, how much did he expend for all?

Arithmetical Formula.—If a farmer purchased broadcloth for $40, a stove for $35, and some milk-pans for $13, he must have expended for all the sum of these amounts, which is $88.

ANALYTICAL STEP.—(c.) Find the amount of money left.

Elementary Question.—If a farmer received $204 and spent $88, how much had he left?

Arithmetical Formula.—If a farmer received $204, and spent $88, he must have had left the difference between these amounts, which is $116.

CONCLUSION—Therefore, if a farmer sold a span of horses for $138, a cow for $38, some butter, &c.

NOTE.—The teacher should require the pupil either to state each *elementary question* distinctly, or, which would be far better, require it written.

EXERCISE

Require the pupil to compose and analyze ten problems consisting of at least three analytical steps. (24:21-2)

When carried to such an extreme, repetition made the statements confusing. This later caused such detailed analyses to fall into disfavor, and by 1870 the method was no longer advertised as a feature of texts.

However, a saner form of the method has prevailed. The authors of the McLellan-Ames series (1898) were among the first to state openly in their text that they felt the wording used in what was known as "logical analysis" was perplexing and tended to increase difficulty rather than make the work clearer. And in 1915, the authors of the *Everyday Arithmetics* commented in the preface that:

> The analysis of problems and the explanations of processes given throughout this book are intended as helps in making the work clear. The practice of the best teachers is opposed to having pupils follow set forms in explaining the solution of problems. (51a:37)

Ever since that time analytical material in texts has been given generally in the form of suggestions and aids; it has been presented where needed rather than only at the beginning of a new topic; and it has been stated clearly and simply. It is still considered a valuable and necessary part of textbook material.

Theory of mental discipline.—Colburn, in 1821, came out with the statement that:

> Few exercises strengthen and mature the mind so much as arithmetical calculations, if the examples are made sufficiently simple to be understood by the pupil; because a regular though simple process of reasoning, is requisite to perform them, and the results are attended with certainty. (9:iv)

The idea of the use of arithmetic in strengthening the mind was new at the time but educators seized upon it immediately as another practical value to be gained from the study.

Thompson (1825), whose book followed closely upon Colburn's was typical of authors who quickly accepted this idea, for he stated that "habits of mental computation have a salutary influence upon the mind, by rendering the memory retentive, and producing a promptness of recollection" (10:iii).

Some texts included exercises purely for the sake of strengthening the mind. Hassler (1826), for example, who believed reasoning was a major objective in the study of arithmetic, included such exercises as the following:

> If no other figures were used but (1) and (0), that is presence or absence of the quantity indicated by any rank or place of figure, the value in each place will always be successively *double* of that in the preceding place, and the whole of the calculation would become a mechanical change of places; so for instance, in this system the following numbers 111101, transcribed into our usual decimal system, would be 32, 16, 8, 4, and 1; or (61.) It will be a very good exercise for the reflection of the scholar to try some of this kind of expressions in different systems. (11:14-15)

For many years, especially in the latter half of the 1800's, authors stressed the value of a study of arithmetic in training the mind.

Perkins (1849) stated that his text was particularly designed "to discipline the minds of those who study it, in the science of Numbers, and to advance them to a higher level of intellectual capability" (19:iii); and the Woodward series (1899) claimed that the study of arithmetic developed pupils in such a way as to make them able to cope with situations later in life.

In the 1890's many of the texts still contained exercises for mental discipline, or "work of a purely disciplinary character." However, some educators had already begun to question the disciplinary value of the subject, especially in the lower grades, as is evident from a statement in the preface of Mecutchen and Sayre's text as early as 1877:

> It has been too much the fashion of late to attempt the development of the reasoning powers in children of tender years. Nature dictates a different course. The perceptive faculties are in full vigor during childhood, whilst the ability to reason is slow of growth, and seldom capable of rapid development early in life without risk of serious injury. (31a:2)

About the turn of the century there was evidently considerable discussion about the relative amount of stress to be placed on form and content, for form had become over-emphasized in the attempt to gain disciplinary value, and a number of educators felt that content was very important. This disagreement led to research studies on the subject and the formal discipline theory began to be dropped from the teaching of arithmetic. In the last 30-year period used in this analysis the disciplinary function of arithmetic was not mentioned in the texts.

Oral and written arithmetic.—In the use of so-called "mental" arithmetic, Colburn was again the leader. In writing his text, he kept the numbers in his problems small and within the comprehension of the very young child, for whom his text was designed. His comment that the pupil "will be able to perform all examples in which the numbers are so small, that the operations may be performed in the mind" (9:viii) led into the idea of including oral or mental arithmetic in texts. The plan was accepted quickly by a number of authors. Thompson (1825) was so taken with the plan that he included three pages of mental arithmetic in the front of his text before he had given the table of contents; but there were others who were not so much in favor of the idea, arguing that all arithmetic is mental.

However, the inclusion of oral and written problems in separate groups was extremely popular for many years. Almost every textbook during the latter half of the eighteenth century contained these two types of problems, variously named "for the slate" and "for the eye," "oral exercises" and "slate exercises," "at sight" and "for dictation," "mental problems" and "written problems," and, as late as 1915, "without pencil" and "with pencil." Throughout the texts, the oral exercises were simpler than the written and led up to the written. In the Woodward series (1899) the authors stated that oral exercises were used to develop speed in calculation while the written ones were to

increase accuracy and understanding of their use in the ordinary affairs of life.

Of late, the separation of oral from written exercises has not been so pronounced. Simple problems still appear first, but they are not referred to as oral, and the written problems grow out of these naturally. The pupil is now left to determine for himself which he can do in his mind and which require the use of pencil and paper.

Number taught through measurement.—The influence of Dewey's theory that number should be presented as the "tool of measurement" was evident in several of the arithmetics published in the late 1890's. The one which adopted and applied the theory most completely, however, was probably the McLellan-Ames series (1898), since McLellan had been co-author of Dewey's *Psychology of Number* and he not only favored the theory but thoroughly understood it. In the preface to the primary book in this series the authors stated that one of the principles on which the book was based was that the child should be taught, immediately upon entering school, to utilize his ability to count. They believed this could best be done through the use of measurement.

In carrying out this idea, counting in connection with measurement was found on the very first page of the text, and concerned the number of inches in a foot and yard and the number of feet in a yard. Before the first lesson (of 22 questions) had ended, the child had encountered problems involving linear measure, weight, liquid measure, time, and money.

The use of measurement is definitely a means for developing concrete situations out of what might otherwise be abstract number relationships. It was one phase of the trend toward making arithmetic live for children, introducing it in real situations and showing in this way the need for an understanding of number in everyday life. While it is no longer the major form used for making number concrete, it still holds a prominent place in the teaching of arithmetic.

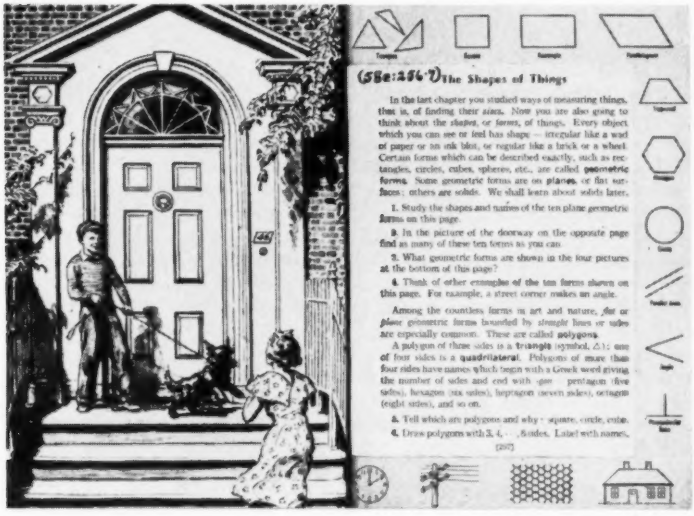
Important New Teaching Theories and Techniques

A number of teaching theories and techniques introduced since 1790 are still gaining in favor and show evidence of continuing to be important factors in textbook construction for many years to come. Almost all of these have as a basis for their use the fact either that they put arithmetic teaching on a more scientific basis or that they make it more interesting or attractive.

Illustrations.—Few illustrations were used in textbooks during the first century of this study, and these few were very simple in form. Now and then a book was "dressed up" with a floral design or conventional pattern at the beginning of each chapter, but (except for Cocker's picture in the front of his text) only two books before 1830—Pike's earliest text (1788) and Thompson's (1825)—used any other type of illustration, and these were very simple line drawings. However, simple geometrical figures and line drawings to illustrate

rules concerning area or other forms of measurement were found in many texts after that date.

Although pictures of schoolroom situations had been found now and then in texts before 1866, French was the first to use illustrations for their human interest value. Many of the pictures in his arithmetic dealt with subjects that appealed to children in their out-of-school life—building bird houses, going to the circus, rolling hoops, etc. His problems were built around pictures, although in only a few instances were there more than one or two problems based on any one illustration. French was definitely ahead of his times in this respect—so far ahead, in fact, that only one other author, White (1870), followed his example before the beginning of the twentieth century. White did not use as many illustrations as did French, but he caught the spirit of adding a touch of fun and human interest to an otherwise uninteresting subject. Typical of the illustrations in his text was one used to illustrate avoirdupois weight. A grocer was weighing a child in the store, with a clerk and a customer looking on amusedly, for the child held an apple behind her back and her pet dog, in his desire to find out what she held, had put his front paws on the scale. Even in recent texts, few pictures have had the appeal of this one, yet it was instructional, too, in that it showed both the large and the smaller scale used in the grocery business. (See Figure 9.)



The Shapes of Things

In the last chapter you studied ways of measuring things, that is, of finding their sizes. Now you are also going to think about the *shapes*, or *forms*, of things. Every object which you can see or feel has shape — irregular like a wad of paper or an ink blot, or regular like a brick or a wheel. Certain forms which can be described exactly, such as rectangles, circles, cubes, spheres, etc., are called **geometric forms**. Some geometric forms are on **planes**, or flat surfaces; others are solids. We shall learn about solids later.

1. Study the shapes and names of the ten plane geometric forms on this page.

2. In the picture of the doorway on the opposite page find as many of these ten forms as you can.

3. What geometric forms are shown in the four pictures at the bottom of this page?

4. Think of other examples of the ten forms shown on this page. For example, a street corner makes an angle.

Among the countless forms in art and nature, *flat* or *plane* geometric forms bounded by straight lines or sides are especially common. These are called **polygons**.

A polygon of three sides is a **triangle** (symbol, △); one of four sides is a **quadrilateral**. Polygons of more than four sides have names which begin with a Greek word giving the number of sides and end with *-gon* — pentagon (five sides), hexagon (six sides), heptagon (seven sides), octagon (eight sides), and so on.

5. Tell which are polygons and why — square, circle, cube.

6. Draw polygons with 3, 4, . . . 8 sides. Label with names.

(three eighths size)

Fig. 9. An example of the use of illustrations for instructional purposes

The introduction of such a frivolous element into textbooks was looked down upon by many authors for years after this, and some even objected to the use of any type of picture because they believed

it would divert the attention of the pupils from the real purpose of the text. However, as the need increased for making a rather difficult subject as attractive as possible to children forced to study it, the value of illustrations became more obvious, and authors in recent years have made a great deal of use of such material. The Smith series (1904) was one of the first of the recent texts to recognize its use other than as instructional material, and stated in its preface that pictures had been used "to show the relations of numbers, to make real their use in measurements, to suggest materials for the use of the teacher, to render more interesting and genuine the various groups of problems, and . . . to present a page that shall attract children . . ." (48a:iv). In this text units of questions, sometimes as much as a page long, were based on the pictures.

New ideas have been introduced into the more recent texts, always with the purpose of adding novelty and up-to-dateness. Stick figures were used in the *Iroquois Arithmetics* (1932), pictures extended to the edge of the page in the last two texts studied, and full-page colored pictures were found in the last three published. Color has been used in all but one of the texts published since 1928. At first only one color was used with black and white, but, in the last three texts, several colors were found in the books designed for the lower grades. In all series fewer illustrations and less color were used in the books for the upper grades and the instructional value rather than the appeal element was uppermost. All recent arithmetics, however, are attractive and their illustrations have added much to their appearance.

On the more practical side in the field of illustrations were the business forms, graphs, and so forth. As has already been said, business forms have been found in the earliest as well as the latest of the texts. At first they consisted only in crude listings of articles on a "bill of parcels," or in the correct wording of a lease or note. The actual business form was not given in these arithmetics, but later an attempt was made to present as accurate a copy of the form as possible, and in the more recent texts engravings have been used, giving an almost perfect reproduction of the business forms studied.

Graphs and scale drawings did not appear in texts until about 1900. The graphs usually dealt with population trends, business trends, and so forth, with problems based on the information pictured, and the scale drawings were often floor plans or landscape designs. Maps, a very useful form of scale drawings, were not introduced until 1928, but have been found in every text since. All these types of practical illustrations are today considered an important part of arithmetic textbooks.

Units of instruction.—It is a little difficult to explain the difference between what is termed a "unit of instruction" in this analysis and what is considered merely a grouping of problems around some common factor. The distinguishing characteristic is that in a unit of instruction the thought that holds the problems together is outside the realm of arithmetic; that is, the problems are based on some historical incident, scientific fact or procedure, business transaction, household or community problem, family situation, or social activity. Recently much

background material has accompanied these units in order to give the child an interest and to make the problems in the unit seem real and alive. This has led at times into a true storybook style.

Problems have always been grouped in one way or another. At first they fell under a certain rule, or dealt with a certain topic, such as measurement. The principle or process involved was the only common element in the problems—each concerned a different situation and authors prided themselves in being original in thinking of different subject matter for each problem.

In these old arithmetics information was given here and there, usually in footnotes, that bordered on arithmetic but was not a factor in actual computation. It was provided to enlighten the student concerning certain facts used in the problems. While this type of thing is not what is today considered a unit of instruction, it was a forerunner of such material and probably led into the modern version of it. In Pike's 1797 text a good example of this kind of information was found:

> Also, the names of the Coins, less than a dollar, are significant of their values.
>
> For the *Mill* which stands in the 3d place, at the right hand of the comma, or place of thousands, is contracted from *Mille*, the Latin for *Thousand*:—*Cent*, which occupies the second place, or place of Hundredths, is an abbreviation of *Centum*, the Latin for *Hundred*:—and *Dime*, which is in the first place, or place of tenths, is derived from *Disme*, the French for Tenths. (5:102)

The first true indication of the idea of using a central theme outside the realm of arithmetic was found in French's *First Lessons in Numbers* (1866). French, as has been said, was far ahead of the times in his use of illustrations, and he was almost as advanced in his idea of giving his problems a connecting thought. He did this with the aid of pictures, often listing a page or more of questions on what was presented in an illustration. Sometimes the questions dealt with a circus, with sledding, or with shopping, and, in his discussion of lines and angles and surfaces, with the panes of glass in a Gothic window. His questions were very simple in his elementary text. For example, under a picture of children playing in a school yard, he asked:

> 1. In this picture we see 3 boys at play, and one third of their number is in the street. How many boys are in the street?
>
> 2. In the picture are also 6 girls at play and one third of their number is in the street. How many girls are in the street? (29a:81)

Few authors accepted this idea until some time after the turn of the century. Here and there a unit would be found, but usually the material seemed to have been organized more for the convenience of teaching certain principles, as in the Southworth series (1893-95) where two or three pages given to problems relating to the buying and selling of land in separate lots and laying out streets and sidewalks were based on a chart of the land.

The *Everyday Arithmetic* series (1915) was the first to use this method extensively. It contained information about the lakes, rivers, and other geographical features of the United States; facts about the habits and migration of birds, furnished by John Burroughs and other naturalists; and stories behind the business proceedings of everyday life, such as discussions of the making of certain commodities, the services rendered by banks to business concerns, and so forth. New subjects were often introduced in narrative form, with the problems growing out of the discussion.

Even before 1915 authors had grouped problems around units relating to farm or community projects, but the problems still were somewhat disconnected. While they related to a certain general topic, each problem was independent of the others so far as forming a total picture was concerned. In later books these have been more closely related—a child's room is decorated, for example, and the purchase of wallpaper, carpet, curtains, furniture, and so forth, for the room serves as a basis for the problems in the unit. Because of the use of this central theme, the pupil often begins to take a personal interest in the room. He can picture it to himself, perhaps even imagine it to be his own room, and what is more, he can see that, in decorating it, he needs a knowledge of arithmetic.

Units have grown larger and larger. At first they consisted of only a few problems, with almost no background material, but their usefulness in teaching has proved so great that authors are now setting up much of their material in this form. School and community, farm and city were the usual subjects in the beginning, but these have branched out into many other fields. Today texts sometimes contain whole chapters with a single central theme. In the *Study Arithmetics*, the entire seventh grade text dealt with the mathematics of the home while the eighth grade book was built around the mathematics of the community.

Of late, authors have been very careful to give correct information in their units, and in this way they have correlated other in-school and out-of-school subjects with arithmetical facts. "Our Disappearing Forests" (55c:83) was a unit in the *Triangle Arithmetics* (1928), "Oyster Farming" (59d:95) was the subject of a section in the *Study Arithmetics* (1936), and, in the *Daily-Life Arithmetics* (1938), pictures of actual newspaper clippings were used, with problems based on the information given in the article. Material of this type is very effective in relating arithmetic to real situations. (See Figure 10.)

The story element, or central theme, of these units has had a strong appeal and has gained the interest of many students who otherwise could see no reason for learning arithmetic. Since it is believed that this characteristic of instructional units is one that has attracted children, textbook writers are beginning to go a step further and to write units in storybook form, weaving the instructional element into it in such a form that the pupil scarcely recognizes the fact that he is learning something new. This type of unit lends itself best to elementary teaching, of course, but is being carried into the upper grades, too, to some degree. One of the best examples of such units was a

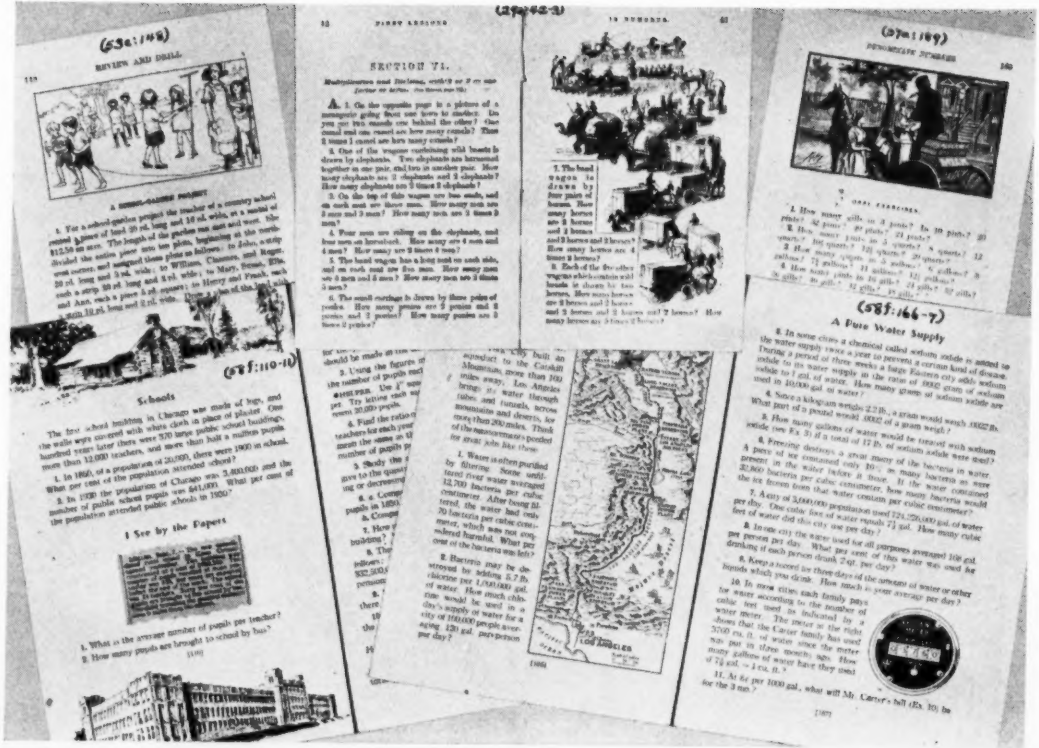


Fig. 10. Typical units of instruction (approximately one third size)

story entitled "Bosco and Bozo," in the third grade arithmetic in the *Study Arithmetics* (1934):

> Patty did not want to go to bed. The clock said it was eight o'clock, but Patty wanted to play with her toy builders.
>
> "Well," said Uncle Jack, "I think your toy builders have a story to tell."
>
> He picked up two spools and five sticks and made a toy boy. Then he made another toy boy like the first one.
>
> How many spools were used to make one boy?
> How many spools were used to make two boys?
>
> How many sticks were used to make one boy?
> How many sticks were used to make two boys?
>
> "These two boys were twins," Uncle Jack began. "They looked so much alike that no one could tell them apart.
>
> "Their names were Bosco and Bozo." (59a:145-6)

The story then continued for several pages and was illustrated throughout with pictures of the boys' dogs, chickens, dog houses, beds, etc., made out of spools and sticks. Problems developed normally out of the situations, becoming almost a part of the story. In this way the child learned to add simple numbers so unconsciously that he scarcely realized he was doing his arithmetic lesson. Is it any wonder that such a method of teaching is gaining in popularity?

In the more advanced books the imagination cannot play quite such an important part, but it, too, has been used to good advantage in the *Study Arithmetics* in teaching such subjects as percentage. A picture of a man at his desk, with his daughter standing beside him, was given at the top of the page. This was followed by:

> The picture above shows Mr. Winters at work on his investment records. His daughter Grace is helping so that he can finish in time to take her to a movie.
>
> Mr. Winters told Grace that he had $3600 invested in a coal mine. His records showed that during the past year this $3600 had earned $144 for him. Mr. Winters wanted to find what per cent the earnings were of the $3600 invested.
>
> He wanted to find the *rate of earnings* for the year.
>
> To find the rate of earnings, he had to find what per cent 144 is of 3600. He did this as shown below. (59e:208)

(At the left of the page here the process was set up mathematically.)

> Mr. Winters first thought, "I made $144 on $3600; 144 is 144/3600 of 3600."
>
> He next thought, "I should change 144/3600 to a decimal fraction; to do this, I must divide the numerator by the denominator."
>
> Explain how he divided.
>
> He then thought, ".04 equals 4%."
>
> What was the rate of earnings?
>
> We can check this answer by finding 4% of $3600. 4% of $3600=$——. How can we tell from this that 4% is correct for the rate of earnings? (59e:208)

The discussion continued in the same vein but concerning another investment of Mr. Winters. Then followed problems involving percentage. By the time the pupil was ready for the problems he understood what percentage meant and how it applied to investments. While this unit was not as different from earlier units as was the story in the third-grade book, yet it showed a distinct effort to fit arithmetic to real situations and to make it live for the pupil.

Review material.—Several of the earliest texts contained nothing in the form of what would today be referred to as review, drills, or test material. Now and then a text boasted of having included a variety of "applicative examples," but these were merely problems used as exercises for the pupils who were working on a certain rule. The nearest approach to review problems was found in lists at the back of some of the books, usually called "promiscuous questions," but these, while touching on almost all the subjects taken up in the text, were usually problems of a more difficult character than those found with the discussion of the subject, and were designed for the use of students interested in more advanced work.

DeMorgan (1832) was the first to call these questions "exercises" and to include them at the ends of the chapters instead of grouping them on the last few pages of the book. This was the first step toward the use of sets of questions for review purposes. A little earlier (1827) Adams had introduced the heading "review," but problems included under that heading had dealt only with rules, definitions, and principles, and were placed at the bottom of the pages on which these questions were answered. For example, "Review of Involution and Evolution" began "What is involution? What are powers? How are the different powers represented? How is a number involved? What is evolution?" (17:274) and so forth.

A number of authors approved of Adams' plan for placing questions in fine print at the bottoms of the pages, but carefully labeling them as to article number so that pupils could find the correct answer. Some also applied his term "review" to these questions. From the point of view of review as we think of it, the questions were of little value, as the pupil was not even called upon to memorize the answers, since they were printed there before him where he could refer to them. Felter (1862) got around this difficulty by basing his "footnote questions" on content learned 100 or more pages earlier in the text. In this way, his questions could be termed more strictly a form of review. However, he was not quite willing to put the student "on his own," so he inserted the article number to which the student could turn for the answer, for example, "Analyze at the blackboard an example in the addition of denominate numbers. (102.,a.)" (24:201) and "What is a quantity? (6.)" (24:78).

Felter's explanation of his method of review was as follows:

> The method of review in this series is such that a pupil entering at almost any stage of progress will be required to review all the subjects over which the class have passed, before finishing the book, thus avoiding the necessity of turning the class back on account of a few of its members. (24:vii)

"Review" became a very popular term, and was adopted by almost every textbook published in the last half of the nineteenth century. Review problems have been a standard part of texts since they were first introduced and have become increasingly popular, although in the last 50 years they have involved the use of arithmetical principles and processes rather than the repetition of rules and definitions.

Sometimes review material appeared at the end of each chapter, sometimes at the back of the book, and sometimes wherever it seemed to the author to be best. There has been no uniformity in placing it —in fact, it has recently been included in several different places within the same text. If placed at the ends of chapters or topics, the problems were based almost wholly on the subject discussed in that division, but if at the back of the book they usually concerned the text as a whole.

By 1895 a more scientific study of "drill" was being made and textbook writers began making suggestions as to how the drill material should be used. The Walsh series, in that year, mentioned the fact that, while drill should be frequent and regular, it should not be carried on too long at a time. The trend since then has been toward including review more often, but giving less at a time.

The spiral method of presenting material has lent itself well to the use of constant review. As each topic was taken up at a more advanced level, the elementary work already covered in the field was reviewed. It was here that the use of review in a form other than questioning made its first important appearance. Since then the great value of constant repetition of principles in helping the student to remember his arithmetic has been proved. In the last few years the term "review" has usually been used to designate additional sections of problems on a principle presented earlier in the text, a practice very common in recent arithmetics.

The publication of textbooks in series has also increased the opportunity to include more review material, for the major points in each year's work were reviewed in the first chapter of the book for the following grade. Review of this type was of course not always labeled as such, but there was evidence that writers had considered it very important.

The problem recently has been to determine what amount of review is advisable, and authors do not as yet seem agreed upon the subject. While there has been a tendency constantly to increase the space given to review, it seems likely that this trend will not continue long. The claim made by the authors of the *Study Arithmetics* that they have maintained a proper balance between review and new material indicates that educators are thinking seriously about the matter.

Drill material.—So far as the tables of contents of these texts reveal, drill sections were not found in arithmetics until 1895. However, material very similar to what is today termed "drill" was found in all the texts analyzed. In some of the earliest arithmetics tables for addition and subtraction, and for multiplication and division were included, and the pupil was drilled on his number combinations. Often long lists of abstract problems were given, sometimes a series

of additions, of fractions to be changed to decimals, or of some other type of problem which, when analyzed, seemed very like what is called drill material in texts today.

Two common forms for tables of number combinations in early texts were (1) the box type, with figures from one to ten across the top and down the left-hand column, and their multiples (or sums, if an addition and subtraction table) in the square below and across from the numbers being multiplied; and (2) the column type, with statements in each column such as "1 and 1 are 2" opposite the column giving "11 and 1 are 12." This latter type sometimes went into numbers above 100. French's (1866) understanding of psychology caused him to present his combinations in the form in which they would most likely be met in everyday life, an idea which is today considered very desirable. In setting up his subtraction tables, for example, he used the following form:

	1	2	3	4	
1	1	1	1	1	etc. (29b:27)
	0	1	2	3	

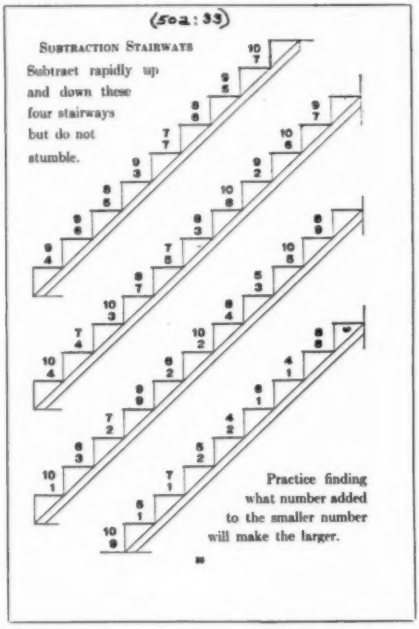
French also included what he called "tabular drills" and gave explicit instructions to the teacher as to how to use such material. Instructions concerning a blackboard drill in addition follow:

> *Tabular Drills.*—1. *For Addition.* Write a column of figures upon the blackboard, as here shown. After the table of 4, page 101, the figures 0, 1, 2, 3, 4, variously arranged, should make up the column; after the table of 5, the column should contain only the figures 0, 1, 2, 3, 4, 5, and so on, of all the tables. The following methods of conducting the exercises will be found valuable:
>
> (1.) Class add in concert, commencing at the foot of the column, and giving results as you point, with an index or pointer, to the numbers: 2, 6, 7, 10, 12, 12, 15, 16, 20.
>
> (2.) Proceed in the same manner, commencing at the top of the column.
>
> (3.) Commence at any place in the column, as at 3, and add up to the top of the column, and then from the bottom, or foot, to the place of starting. Thus, 3, 5, 5, 8, 9, 13 (now at the foot of the column), 15, 19, 20.
>
> (4.) Commencing at the same place, add in the opposite direction.
>
> (5.) Call out a pupil to add before the class, in any of the four methods above described, and require the class to correct his errors.
>
> (6.) Let any pupil, with index in hand, point to the numbers, in any of the orders above enumerated, and the class add in concert.
>
> (7.) In the same manner, let a pupil point to the numbers, and the other pupils, each in turn, name a result. (29a:119)

4	
1	
3	
0	
2	
3	
1	
4	
2	
—	

Drill tables and exercises were especially popular in the latter part of the nineteenth century, when the disciplinary value of arithmetic was stressed. Many original ideas were introduced for the sake of lending variety. In the Southworth series (1893), for example, were two columns of 23 figures each, five columns wide. These were lettered alphabetically. The pupil was asked to add numbers, without

copying, from *p* to *y*, for instance, or from *c* to *r*. Thus an almost inexhaustible number of combinations could be practiced with the use of only two columns of figures. Speed had become an important factor by this time, too, and it was suggested in this lesson that the pupil do each problem in one minute. (See Figure 11.)


(502:33)
SUBTRACTION STAIRWAYS
Subtract rapidly up and down these four stairways but do not stumble.
Practice finding what number added to the smaller number will make the larger.

(approximately one half size)

Fig. 11. One method used for making drill on the fundamentals more attractive

As educators began to realize that some number combinations were much more difficult than others, they varied the square box table slightly, setting the harder combinations in bold-face type to call attention to them. This finally led to the omission of drill tables from most texts, the emphasis being placed only on the combinations which children most often failed to remember. There has been disagreement among authors as to which combinations should be emphasized most, as there has been, and still seems to be, concerning the number of essential number facts.

Drill material is still an important part of arithmetics, but it appears in recent texts in much more attractive forms. Both the *Progressive Arithmetics* (1924) and the *Study Arithmetics* (1934-37) used what they called "Blackboard Ladders," the numbers being placed on the rungs, and in the later grades drill was sometimes referred to

as "Warming Up Exercises" (59d:26). In the *Child-Life Arithmetics* (1936-37) drill was called "Helping You to Know" (57b:48). When drill can be made entertaining it will serve its purpose much more effectively, and it seems the trend today is toward making it as interesting as possible.

Tests.—Tests are comparatively new as a type of content for textbooks. The "promiscuous" questions or "questions for examination" found in early arithmetics were included either as review exercises or as additional work to be assigned if the teacher saw fit. In Colenso's text (1864) were about 40 pages of examination questions quoted from tests given by the New York, Philadelphia, New Jersey, etc. schools. Whether these were given solely as an aid to the teacher or whether they were included for the purpose of showing teachers what others in the field were doing is not known. However, if the latter was the purpose, it was the first indication of an attempt to standardize the subject.

Progress tests, found in Hagar's text in 1871, were the first evidence of an attempt to test the pupil on what additional material he had learned since he was tested previously. However, real testing as it is known today did not enter into texts until much later. The testing movement which has swept the country since early in the twentieth century was started by Rice in 1902 and carried into the field of arithmetic by Stone and Courtis (1908 and 1909). It was welcomed by educators as a progressive movement and almost immediately was included as a distinct feature in their textbooks.

With the idea promoted at first that it was necessary to know a child's I.Q. in order to help him in his arithmetic, some of the texts included mental or alertness tests. However, these were soon dropped and only tests dealing directly with arithmetic were included. At first only the survey or inventory type of test was used, but in 1928 the *Triangle Arithmetics* introduced the diagnostic test into textbooks and since that time these tests have been a special feature of most arithmetics, as teachers and pupils alike have found them valuable in determining the specific difficulties of pupils and, where remedial tests are also given, in helping them to overcome their difficulties. In some of the arithmetics these tests were standardized, in others not, but both types have been helpful in giving both the pupil and the teacher an idea of the pupil's achievement.

Providing for individual differences.—While there is a range of achievement within which most children of a certain age may be expected to fall, there are always some whose achievement is above or below that of others, and a few are so different in ability as to demand special teaching.

Educators have tried one means after another of coping with these differences when pupils are taught in classes grouped, to a great extent, according to chronological age. Gough, as far back as 1801, suggested dividing the class into two or three parts on the basis of individual progress. This has since become a common practice, but it has not been thoroughly successful because it means pupils are still taught

in groups where the class average determines the amount of material taught and the rate at which it is presented.

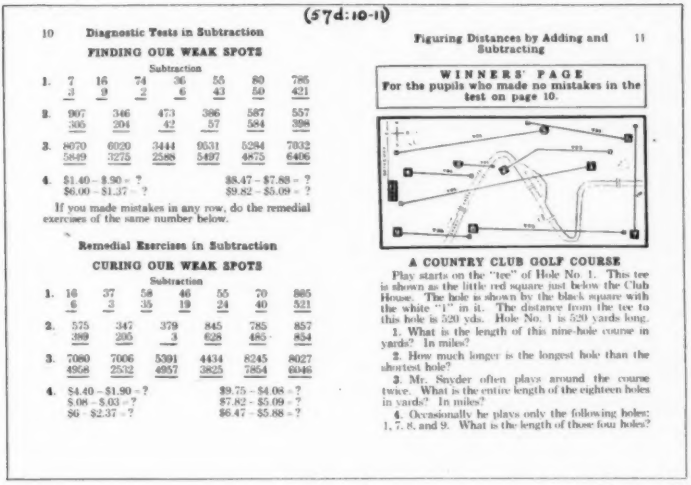
Little further evidence of an attempt to provide for individual differences was found before 1915, unless, perhaps, the progress tests in Hagar's text, were used for this purpose. In the last 30-year period covered by this analysis, however, great interest has been taken in the individual pupil. Almost every text has stressed in its preface that it has made provision for individual differences and, in the *Child-Life Arithmetics* (1936-37), this was elaborated upon with the statement that the adjustment had been made "by a scientifically organized program of diagnostic testing with remedial exercises" (57a:iii).

About 1920, when minimum essentials in various fields were being stressed, the *School Arithmetics* and *Progressive Arithmetics* contained sections on "Minimum Essentials." In these a standard was set below which no pupil could fall and still pass the course. The knowledge required was, of course, very simple and consisted almost entirely of drill material. In the *Modern Life Arithmetics* (1928) a different kind of provision for individual differences—the project method—was used. This was similar to minimum essentials except that the pupil was assigned a project and the projects could vary in difficulty according to the ability of the pupil.

After the diagnostic arithmetic test made its appearance in textbooks, remedial or instructional tests soon followed. These were used to help the pupil overcome his difficulties and to check on his success in doing so. In this connection the word "test" was often avoided because in some children's minds the term was associated with unpleasantness and even fear. In the *Child-Life Arithmetics*, for instance, diagnostic tests were called "Finding Our Weak Spots" and instructional tests were titled "Curing Our Weak Spots" (57d:5). The remedial tests usually opened with some such statement as: "If you made mistakes in any exercise on page 150, do the exercise of the same number on this page" (57d:151). The pupil thus received additional practice on material with which he was having difficulty. This textbook made still further provision for individual differences by including here and there a "Winner's Page," for those who had made no mistakes on a certain page earlier in the book. Questions on this page usually called for ability to follow directions accurately as well as to think through the means of solving a problem. (See Figure 12.) An example of questions found on this page is:

> If you would add, write A after 1.
> If you would subtract, write S.
> If you would multiply, write M.
> If you would divide, write D.
>
> Do the same after each number on your paper.
>
> 1. If you know how tall you were on your last birthday and how tall you are now, how can you find out how much you have grown since your last birthday? (57c:64)

Provision for individual differences has gained the approval of almost all educators and is found in most texts today. The stigma



(57d:10-11)

10 **Diagnostic Tests in Subtraction**

FINDING OUR WEAK SPOTS

Subtraction

1.	7	16	74	36	55	80	785
	3	9	2	6	43	50	421
2.	907	346	473	386	587	557	
	305	204	42	57	584	398	
3.	8670	6020	3444	9031	5284	7032	
	5849	3275	2588	5497	4875	6406	

4. $1.40 − $.90 = ? $8.47 − $7.88 = ?
   $6.00 − $1.37 = ? $9.82 − $5.09 = ?

If you made mistakes in any row, do the remedial exercises of the same number below.

Remedial Exercises in Subtraction

CURING OUR WEAK SPOTS

Subtraction

1.	16	37	58	46	55	70	885
	6	3	35	19	24	40	521
2.	575	347	379	845	785	857	
	389	205	3	628	485	854	
3.	7080	7006	5391	4434	8245	8027	
	4968	2532	4957	3825	7854	6046	

4. $4.40 − $1.90 = ? $9.75 − $4.08 = ?
   $.08 − $.03 = ? $7.82 − $5.09 = ?
   $6 − $2.37 = ? $6.47 − $5.88 = ?

Figuring Distances by Adding and Subtracting 11

WINNERS' PAGE
For the pupils who made no mistakes in the test on page 10.

A COUNTRY CLUB GOLF COURSE

Play starts on the "tee" of Hole No. 1. This tee is shown as the little red square just below the Club House. The hole is shown by the black square with the white "1" in it. The distance from the tee to this hole is 520 yds. Hole No. 1 is 520 yards long.

1. What is the length of this nine-hole course in yards? In miles?

2. How much longer is the longest hole than the shortest hole?

3. Mr. Snyder often plays around the course twice. What is the entire length of the eighteen holes in yards? In miles?

4. Occasionally he plays only the following holes: 1, 7, 8, and 9. What is the length of those four holes?

(three eighths size)

Fig. 12. Provision for individual differences

that used to be attached to work given for the slow student is being overcome to a great extent by the use of such titles for diagnostic and remedial tests as those used in the *Child-Life Arithmetics*. They carry the idea beyond that of merely "getting by," that was intimated by the term "minimum essentials," to the level of an incentive to help oneself and try to accomplish more.

For those who had completed their other work, one or two arithmetics gave tests containing catch questions that called for extreme care in reading. The *Study Arithmetics* also included a "progress chart" so that the pupil could keep a record of his achievement. In every new text that appears original ideas are being introduced to help the individual student in the learning of his arithmetic. With such encouragement he stands a much better chance of succeeding.

The personal element.—Along with the desire of textbook writers to be as helpful as possible to the student, there is an intangible something about the new texts that is closely linked with their appearance, their content, and their method, yet is truly not any one of these alone—a quality that might be termed the personal element. In a way, this is a new approach to the subject and therefore is discussed here as method. It has grown out of the changes that have taken place in texts in making them more readable and more appealing. Simplification has played its part; grade leveling has been a step forward; the correlating of arithmetic with other subjects, the presenting of material in storybook form, and the attempt to socialize arithmetic and to relate it to everyday happenings have all shared in developing this personal element.

Early authors tried to create a feeling of close relationship between teacher (or textbook) and pupil by using the first or second person instead of the impersonal "it" or "they." But a "must" attitude always accompanied the work. Rules and definitions were to be memorized; examples were given with each step set out as a requirement with "When . . . is true, do . . . ," and "Let it be required to . . . ."; and the problems were often stated "I demand to know . . . ." or "Pray tell me . . . ." The child was continually kept conscious of the great learning of the master, the vast knowledge open to him through the text, and the enormous gap between what he knew and what he should know.

Many things have entered into the changing of texts from this dictatorial tone to a friendly, helpful one. The factors leading up to the introduction of the personal element cannot be traced in detail here because such a discussion would involve almost every trend in arithmetics through the years. Changes in the wording of problems, in the presentation of new material, and even in the physical make-up of the books have been evident in the pictures and examples given elsewhere in this study. These have all played a part in making texts more personal.

Perhaps the two factors which contributed most to this characteristic of modern arithmetics were the simplification of statement and of content so that the text was on the child's level, and the use of subject matter with which the child was acquainted. In this way the pupil met everyday problems in their natural environment. The following, from the *Iroquois Arithmetics* (1932), is an illustration of a practical example (showing how to multiply dollars and cents) in which the thinking rather than the rule is emphasized in the solution of the problem:

> A farmer sold four barrels of apples at $4.95 a barrel. How much was he paid for the apples? We can find what he was paid by multiplying.
>
> $4.95 Think 4 x 5=20. Write 0. Carry 2.
> 4 Think 4 x 9=36. Add 2. Write 8. Carry 3.
> ——— Think 4 x 4=16. Add 3. Write 19.
> $19.80 The farmer was paid $19.80 (56a:98)

The problem used in this example is typical of both the subject matter and the form of expression in problems found in the majority of recent arithmetics. It is clear and straightforward in statement, simple in vocabulary, and familiar in subject matter. The question asked is reasonable from the point of view of the child and of business practice, and the explanation that follows is designed to show him how to think through the problem step by step.

Rules and definitions in these texts were introduced as suggestions or helpers, such as:

> Remember, 0 means not any, or no. (54a:45)
>
> Dollars and cents are subtracted just as other numbers are subtracted. When you write them be sure that the decimal points are under each other. (56a:66)
>
> Omit 0's in division when you do not need them, but keep the 0's if you would make mistakes without them. (58b:221)

When rules were presented in this way the pupil was given the impression that someone cared whether he did his lesson well, and was there on the sidelines to offer a hint or reminder when he needed it. The same atmosphere was created in introducing short cuts. Instead of including them merely as more material to be learned, the *Child-Life Arithmetics* gave them such headings as "A Trick that Helps" (57c:71). When presented in this way, the emphasis was placed on the advantages of knowing certain things rather than simply on the subject itself.

The *Study Arithmetics* carried this same approach throughout the text. This series had an unusually conversational tone in the problems as well as in the explanatory material. It was written as if spoken by one of the group, not as if planned by adults for children. There is little doubt that this method will be used more and more, for it has been increasingly popular in the last 15 years. It is not based on play or story telling, although these are a part of it—it is a quality that emanates from a real personal interest on the part of the textbook writer in the welfare of the pupil.

Adjustment to Grade Level Needs

The problem of adjusting textbook material to the pupil's level arose soon after texts began to be used in schools, for the older arithmetics were completely beyond the comprehension of the child of school age.

Daboll (1799) was the first to state that his text was written for schools and that he had therefore tried to make his rules as "concise and familiar" as possible. Although his efforts were spent mostly on rearranging subject matter, it was a step toward writing texts for younger people rather than for adults.

Colburn was the first to succeed in designing a text especially for children and in limiting himself to language which could be understood by those of primary school age. His *Intellectual Arithmetic* (1821) pointed the way, and there was evidence from then on of a sincere desire on the part of textbook writers to simplify their texts and to use material within the range of the ability and interest of children in the grades for which the text was planned.

At first there were some who objected to making arithmetic easier for the pupils, for they claimed, as did Talbott (1833) that "*labor* and *labor only*, can insure success in any pursuit" (16:4). However, educators began to agree more and more with the point of view of those who were trying to make the subject more understandable and more interesting.

Simplification of topics.—In the simplification of topics two major types of change took place. First, very difficult or seldom used topics, such as alligation alternate, permutations and combinations, and trigonometry, were either advanced to higher mathematics or were discarded entirely. Second, unnecessary material within topics was omitted; that is, the phases of the subject which were so remote from daily life as not to be needed by the average person were omitted.

In the early arithmetics every possible detail concerning a subject was included. In Pike's (1788) discussion of numeration, for example, the different numbers and their names and relationships were shown in a chart characteristic of the times. The chart and its explanation were based on the English way of counting, a numbering system used in most of the early American arithmetics. Hung on this diagram was a footnote designed to take care of the needs of those dealing in still larger quantities, to the effect that:

> Note 1: Billions is substituted for millions of millions: Trillions, for millions of millions of millions: Quatrillions, for millions of millions of millions of millions.
>
> Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, &c. answer to millions so often involved as their indices respectively denote. (3:18)

Is it any wonder that children were lost in a maze of numbers when they encountered something like this within the first few pages of their texts? In their daily experiences they seldom used numbers of more than 100, and such large figures as were given in this footnote were meaningless to them. Colburn saw the error of confronting the beginner with such a complete and technical discussion and opened his text, as has already been stated, with the counting of fingers and thumbs.

In the more recent texts the topic has continued to be presented in this informal fashion. The study of the larger numbers has not come until later, after the pupil was thoroughly grounded in the meaning of number.

Many other topics have gone through the same process of simplification and personalization. In some cases, principles or processes, seldom used in everyday life but needed sufficiently often for educators to believe they should be included in arithmetics, have been simplified to the extreme. Perhaps the clearest example of this in the textbooks studied was in the method taught for extracting square root.

Square root was a topic found in almost every textbook throughout the 150 years. In all but seven texts before 1890 the study of powers and roots was referred to as involution and evolution, but after that date none of the arithmetics used this more technical connotation. In Gough's *Practical Arithmetic* (1801), the rule, a puzzle in itself for most children, was written in rhyme, making it still more difficult to understand:

> First to prepare the Square, this do;
> Point off the Figures two by two:
> Beneath the last the Square next less
> Put; and its Root i' th' Quotient place:
> From the last Period take the Square,
> Then the next lower Period there
> To the Remainder must be brought;
> Be this a Dividend: the Quote
> Doubled must the Divisor be
> To all but Units' Place; then see
> How oft the greater holds the less,
> That Figure must the Quote express,

> And the Divisor Units too,
> Then as in plain *Division* do.
> Thus every Period one by one
> We manage, and the Work is Done. (4:285)

Fortunately, other authors did not take up with Gough's poetic style in giving this rule. Most of them divided the process into steps and presented the steps in order, the discussion being clear in so far as a very formal, technical explanation far removed from one's own experiences can be made clear. In some of these, the authors attempted to handle the subject very thoroughly. Greenleaf (1842), for example, after a careful and clear explanation of the rule, added seven notes explaining exceptions to the rule and unusual situations that might arise.

Many authors gave examples, worked out on the page opposite the rule, and some gave a geometrical explanation, including a line drawing to help clarify the work. Ray (1879) gave one of the clearest explanations of this kind, taking up each step as if it were a thought process rather than a hard, cold rule. Algebraic equations were also, for a few years, a popular means used to assist the student in seeing the true meaning of square root.

Since the beginning of the twentieth century, the discussion of square root has taken up less and less space in textbooks. The *School Arithmetics* (1919) suggested factoring as a simple method of finding square root, and, a little later, the *Modern Life Arithmetics* (1928) offered a method of estimating the square root of a number as between the square roots of the nearest perfect square on either side of the given number. While this method supposedly was presented as a "short cut," it became quite involved before being completed.

One or two books gave a short as well as a long form of finding the square root but, by the end of the 150 years covered by this study, only a table of squares and square roots (of numbers from 1 to 120) was included. No rules for extracting square root were given, but the pupil was instructed in the use of the table for determining either the square or the square root of any given number within the limits of the table.

Simplification of rules and definitions.—All through the early texts analyzed in this study rules and definitions took up at least one fifth of the book. During the first 30 years covered, the Rule of Three was the "high point" in the arithmetical learning of the average person. According to *Cocker's Arithmetick:*

> The *Rule of Three* (not undeservedly called the *Golden Rule*) is, that by which we find out a fourth *Number*, in *proportion* unto *three* given *Numbers*, so as this *fourth Number* sought may bear the same Rate, Reason, or Proportion to the *third* (given) *Number*, as this *second* doth to the *first*, from whence it is also called the *Rule of Proportion.* (1:102)

(Here is an extreme example of the overuse of italics and capitalization in early texts.)

Headings in some of the older arithmetics included four different rules—the Single Rule of Three Direct, the Single Rule of Three

Inverse, the Double Rule of Three Direct, and the Double Rule of Three Inverse. One form or another of the rule was used in solving or proving the answers to more than half the types of problems contained in the texts.

An example of the use of this rule in a problem which would today be solved by algebra is the following, found in Daboll's text (1799) under "position," a "rule which discovers the true number by the use of false, or supposed numbers."

> A schoolmaster being asked how many scholars he had, said, If I had as many more as I now have, half as many, one-third and one-fourth as many, I should then have 148; How many scholars had he?

Suppose he had 12	As 37: 148 :: 12: 48	Ans.
as many = 12	48	
½ as many = 6	24	
⅓ as many = 4	16	
¼ as many = 3	12	
Result, 37	Proof, 148	(8:201)

Another rule included in all texts of the early nineteenth century was called "practice." This would today be classified more as a table than as a rule, as it gave the comparative values of different rates of exchange and of different quantities of goods. The data included depended upon the type of problems being studied. In Guthrie's book (1810), these rules were expressed in an extremely formal manner, with a vocabulary apparently designed for the educated adult:

> These Rules are most compendiously contrived, for the speedy casting up of the prices of all sorts of goods, or merchandize; and for the quick despatch of business, and frequent use, are called RULES of PRACTICE.
>
> *Note.* In order for working expeditiously, it will be necessary to understand perfectly the following Table: (14:71)

Here was inserted a "Practice Table" of Federal and English money showing equivalent fractional parts of dollars and cents and of pounds, shillings, and pence, such as 16 2/3¢=1/6 dollars, followed by a statement of methods of proving answers to problems based on "practice."

These were common rules, found in almost all early texts. Many others, stated in about the same form, were included in these arithmetics. In a few books there were also rules covering subjects far removed from the experience of the children studying the text, such as ullaging of standing and lying casks, determining the power of a lever, finding "the tome of the moon's southing," and so forth—things which no child and few adults would ever have occasion to use.

Now and then a rule was given which seemed to be a deliberate attempt to "stump the experts." The juggling of figures for the pure joy of the mental manipulation involved seemed to be the only excuse for two or three of the rules found. One of the best examples of this kind was found in Walkingame's text (1751)—a rule for "multiplying several figures by several, and the product to be produced in one line only."

RULE. Multiply the units of the Multiplicand by the units of the multiplier, setting down the units of the product, and carry the ten; next multiply the tens in the multiplicand by the units of the multiplier, to which add the product of the units of the multiplicand multiplied by the tens in the multiplier, and the tens carried; then multiply the hundreds in the multiplicand by the units of the multiplier, adding the product of the tens in the multiplicand multiplied by the tens in the multiplier, and the units of the multiplicand by the hundreds in the multiplier; and so proceed till you have multiplied the multiplicand all through, by every figure in the multiplier.

EXAMPLES.

Multiply — 35234	35234
By — 52424	52424
1847107216	140936
	70468
	140936
	70468
	176170
	1847107216

EXPLANATION.

First, $\overline{4 \times 4}=16$, that is, 6 and carry 1. *Secondly*, $\overline{3 \times 4}+\overline{4 \times 2}$, and 1 that is carried is 21; set down 1 and carry 2. *Thirdly*, $\overline{2 \times 4}+\overline{3 \times 2}+\overline{4 \times 4}+2$ carried $=32$; that is 2 and carry 3. *Fourthly*, $\overline{5 \times 4}+\overline{2 \times 2}+\overline{3 \times 4}+\overline{4 \times 2}+3$ carried $=47$; set down 7 and carry 4. *Fifthly*, $\overline{3 \times 4}+\overline{5 \times 2}+\overline{2 \times 4}+\overline{3 \times 2}+\overline{4 \times 5}+4$ carried $=60$; set down 0, and carry 6. *6thly*, $\overline{3 \times 2}+\overline{5 \times 4}+\overline{2 \times 2}+\overline{3 \times 5}+6$ carried $=51$; set down 1, and carry 5. *Seventhly*, $\overline{3 \times 4}+\overline{5 \times 2}+\overline{2 \times 5}+5$ carried $=37$, that is 7, and carry 3. *Eighthly*, $\overline{3 \times 2}+\overline{5 \times 5}+3$ carried $=34$; set down 4, and carry 3. *Lastly*, $\overline{3 \times 5}+3$ carried $=18$; which being multiplied by the last figure in the multiplier, set the whole down, and the work is finished. (2:168)

Such rules as these were interesting, without doubt, but they were not useful, and when authors began to plan their texts for the common man and his needs, such material was omitted.

Definitions in early arithmetics were stated in much the same way as were rules—they were very formal, very technical, and very far removed from anything the school children met in daily life. Often, too, they attempted to be all-inclusive, and sometimes treated a topic so extensively as to add material not a part of the term being defined. Daboll, for example, threw in definitions of gross weight, cloff, and suttle when he attempted to define tare and trett:

> Tare and trett are practical Rules for deducting certain allowances which are made by merchants, in buying and selling goods, &c. by weight; in which are noticed the following particulars:
>
> 1. *Gross weight*, which is the whole weight of any sort of goods, together with the box, cask, or bag, &c., which contains them.
>
> 2. *Tare*, which in an allowance made to the buyer for the weight of the box, cask, or bag, &c. which contains the goods

bought, and is either at so much per box, &c.—or at so much per cwt. or at so much in the whole gross weight.

3. *Trett*, which is an allowance of 4 lb. on every 104 lb. for waste, dust, and &c.

4. *Cloff*, which is an allowance made of 2 lb. upon every 3 cwt.

5. *Suttle*, is what remains after one or two allowances have been deducted. (8:14)

By 1875 the results of the efforts of textbook writers to make arithmetics simpler were very evident. In Hagar's *Common School Arithmetic* (1871), for example, the rule for compound interest was direct and to the point, and was stated in a vocabulary that would be known by a child who had progressed that far in his study of arithmetic:

Find the amount of the given principal for the first interval, at the given rate.

Make that amount a principal for the second interval, and so on.

From the last amount subtract the given principal, and the remainder will be the compound interest. (30:216)

And in Milne's text, the same year, examples were used to help clarify the definitions, as:

A *Concrete Number* is a number used in connection with some specified thing.

Thus, 5 books, 7 trees, 8 horses, are concrete numbers.

An *Abstract Number* is a number that is not used in connection with any specified thing.

Thus, 5, 7, 8, are abstract numbers. (32:157)

The idea of using examples in explaining rules and definitions was not new. But it had been found so invaluable in that it presented a fact from more than one angle, that it was and still is used by a great many textbook writers.

Early in the twentieth century authors began to use the principle of working from the known to the unknown in stating their rules and definitions—a method long used for presenting new material in textbooks. In early arithmetics definitions always began "a fraction is . . ." or "discount is . . ."; and rules were just as bad—"the Rule of Three teacheth . . . ," and so forth. The pupil could do little with such statements except to memorize them unless he was mentally capable of reversing them and relating the term to the knowledge he already possessed. It seems strange that so many years elapsed after Colburn first introduced the idea of leading the child from what he already knew to what he was to be taught before this principle was applied to textbook content other than new topics. When it finally came into use it was found to be very effective in simplifying the statement of rules and definitions. In the recent texts the known is mentioned first in such material:

The number in a fraction which tells how many equal parts or groups there are is the denominator. (58b:100)

When we divide both a numerator and a denominator by the same number, we say we cancel. (59d:59)

Recently several authors have attempted to make definitions clearer by giving a certain amount of informational background which would help enlighten the student as to the meaning of the definition. An example of this was found in the *Modern Arithmetics* (1913):

> In United States money a *cent* is the hundredth part of a dollar. The word *cent* comes from *centum*, which means hundred.
>
> The phrase *per centum*, or *per cent*, means *by the hundred, at the rate of*, or *per hundred.*
>
> The whole of any quantity or thing is 100 per cent of it, or 100/100 of it.
>
> Thus, if a man has 800 sheep, 100 per cent of them represents all of his sheep.
>
> Then 200 sheep, or ¼ of them, are ¼ of 100 per cent of them, which is 25 per cent of them.
>
> Instead of writing the words *per cent*, we generally use the sign, (%), which is read "per cent." (50b:202)

This idea of giving a brief account of the reasons why certain terms are used led into the relating of new material to the experiences of the child. This had been done to a limited extent ever since the time of Colburn, but in most cases the material used was connected with some other learning process or with stereotyped experiences such as cutting an apple into halves or fourths in the learning of fractions. While these methods had their place in the teaching of arithmetic, they did not add much life or interest to the subject.

Very few examples of an attempt to clarify material in this way were found before the last 30-year period covered by this study. However, an outstanding one was given in Ostrander's text, as far back as 1829:

> A spheroid is a solid body, like an egg, both its ends are the same. (13:164)

How much clearer this was than one given more than 40 years later:

> A spheroid, or ellipsoid, is the solid described by the rotation of an ellipse about either its longest or its shortest diameter. (27:308)

The child could grasp the idea quickly from Ostrander's definition and, though he might not know the principles back of it, he would be able to recognize a spheroid if it were presented to him. On the other hand, the child learning the second definition would have to be acquainted with the meaning of the terms "described," "rotation," "ellipse," and "diameter" before he could begin to unravel the statement and get its meaning.

Sometimes, in more recent books, a picture has been added to clarify material. For example, in the last book in the *Study Arithmetics* (1937) a picture of a water tank was placed beside the definition of a cylinder, and the cylinder was described thus:

> *A cylinder is any object like this tank whose base and top are circles that are equal and parallel.* (59f:344)

Rules and definitions have not been so important a part of texts recently as they were in the early part of this study. They are stated

today only when necessary for the understanding of the work, and then are given in as clear and simple a form as possible.

Simplification of tables of measures.—One other phase of content in arithmetics, tables of measures, showed the effects of the policy of simplifying and clarifying material so startlingly as to make it especially interesting in this connection.

While tables of measures have been found in almost all the arithmetics, there was a vast difference between the material included in the old texts and that found in the new. In the early years of this study, almost every conceivable field was covered, with sometimes 15 to 20 different measures being presented in a single textbook. Before 1820 many of the texts included Troy, avoirdupois, and apothecaries weights, and cloth, cheese and butter, long, land or square, wine, ale and beer, dry, time, cubic, and "circular measure" or motion. One or two also included what they called "solid measure," a measure used for hewn timber, a "miscellaneous table," and a "Table of the Measure of Length of the principal places in Europe, compared with the American yard" (3:386). Even as late as 1878 very technical tables were also found in some of the texts, such as diamond weight, mariner's measure, and commercial weight.

However, in several of the older texts included in this study, some of these tables were being combined. Apparently this change came as a result of business practice, which found little need for so many different types of measures.

Wine and ale and beer measures were among the first to be combined. Guthrie's text (1810) presented these as "liquid measure," and below the table, which included pints, quarts, gallons, barrels, hogsheads, pipes, tuns, tierces, and puncheons, the author explained that "By this measure all sorts of Wines, Spirits, Brandies, Ciders, Beer, Ale, Oil, Honey, &c. are measured. Note.—231 Solid Inches make a wine gallon, and 282 solid inches make a gallon of beer or ale" (14:8). It seems to have taken a little time for wine merchants and beer and ale merchants to agree to give the same amount in a gallon.

For a number of years after this wine and beer measures were still carried in many of the texts as two separate measures. Usually an explanation of their different uses accompanied them. Dodd (1849), for example, stated that wine measure "is used in measuring *wine*, and, in general, all *liquids* excepting such as fall under Beer Measure" (21:127), while beer and ale measure "is used in measuring *beer*, and other *malt liquors*, and, in some places, milk and water." Seldom was milk mentioned anywhere else in these early texts, yet beer, wine, etc. were the subject content of a large number of the written problems. Recently the emphasis has been reversed, with milk a popular subject and spirits of all kinds tabooed.

The first indication of a combination of cloth, land, and long measure in one table, called linear measure, came a little earlier than this, when Adams (1827) stated that "cloth measure is a species of linear measure, being used to measure cloth and other goods sold by the yard in length, without regard to the width. The denominations are ells, yards, quarters, and nails" (17:139). The tables which

accompanied this explanation included the different numbers of quarters (equal to 9 inches or 4 nails) needed to make up a Flemish ell, an English ell, and a French ell. Such detail must have served to confuse the pupil without giving him information he could use outside the schoolroom.

Now and then in the older texts a table of miscellaneous linear measure was given which included hands (used in measuring the height of horses at the shoulder), spans, paces, fathoms, miles, leagues, geographical and statute miles, and degrees. The linear measure in common use today includes inches, feet, yards, rods, and miles, a table made up mostly from the former land measure, with barleycorns (three to the inch) and furlongs being omitted.

Many of the older books contained what was called a miscellaneous table—a sort of "catchall" for information that did not conveniently fall within one of the other groups of measures. Perhaps the most complete of these was found in Adams' book (1827). It is quoted here to show how many kinds of information worth little to the average person were given in tables that the child was expected to memorize.

Miscellaneous Table

20 units	make 1 score
5 score	1 hundred
12 units	1 dozen
12 doz.=144	1 gross
12 gross=144 doz.	1 great gross
200 lbs. of beef, pork, or fish	1 barrel
196 lbs. of flour	1 barrel
8 bushels of salt	1 hogshead
280 lbs. of salt at the salt works in N.Y.	1 barrel
100 lbs. of raisins	1 cask
100 lbs. of fish	1 quintal
100 lbs.	1 hundred
18 inches	1 cubit
22 inches, nearly,	1 sacred cubit
1 gallon of train oil	7½ lbs.
1 gallon of molasses	11 lbs.
24 sheets paper	1 quire
20 quires	1 ream
2 reams	1 bundle
5 bundles	1 bale

A sheet folded in two leaves, or 4 pages, is called a folio
A sheet folded in four leaves, or 8 pages, a quarto, or 4to
A sheet folded in eight leaves, or 16 pages, an octavo or 8vo
A sheet folded in twelve leaves, or 24 pages, a duodecimo, or 12mo.
A sheet folded in 18 leaves, or 36 pages, an 18mo.
A sheet folded in 24 leaves, or 48 pages, a 24mo.

6 points	make 1 line,	Used in measuring the length of
12 lines	1 inch,	the rods of clock pendulums.
4 inches	1 hand,	used in measuring the hight of horses.
6 feet	1 fathom,	used in measuring depths at sea.

(17:147)

Adams also went into great detail in his table of time, which contained seconds, minutes, hours, and days in a week. Then, instead of being practical and saying there were 52 weeks in a year, he became

very precise, saying "52 weeks, 1 day, 5 hours, 48 minutes, 48 seconds, . . . make 1 year" (17:145). He followed this with an explanation of "Bissextile, or Leap Year." For some reason he omitted months from the table—perhaps a 12-month year was even more difficult to explain.

No other author had been so exacting in his explanation of the length of a year since Cocker, more than a century and a half earlier, who went into great detail, but did not always agree with Adams' statements. Cocker accompanied his table with the following discussion of the months of the year:

> But the Year is usually divided into 12 unequal *Kalendar Months* whose Names and the number of Days they contain, follow, *viz.*
>
> | | Days |
> |---|---|
> | *January* | 31 |
> | *February* | 28 |
> | *March* | 31 |
> | *April* | 30 |
> | *May* | 31 |
> | *June* | 30 |
> | *July* | 31 |
> | *August* | 31 |
> | *September* | 30 |
> | *October* | 31 |
> | *November* | 30 |
> | *December* | 31 |
>
> So that the Year containeth 365 Days, and 6 Hours; but the 6 Hours are not reckoned but only every 4th Year, and then there is a Day added to the latter end of *February*, and then it containeth 29 Days, and that Year is called Leap-year, and containeth 366 Days.
>
> And here note, that as the Hour is divided into 60 Minutes, so each Minute is subdivided into 60 Seconds, and each Second into 60 Thirds, and each Third into 60 Fourths, &c.
>
> The Tropical Year by the exactest observations of the most accurate *Astronomers* is found to be 365 Days 5 Hours, 49 Minutes, 4 Seconds, and 21 Thirds. (1:18-19)

When arithmetics contained such detailed information they were the true reference books they were intended to be. Textbook writers have recognized the fact that scientific information is not necessary to a working knowledge of the measurement of time, and therefore less and less material of this kind has been found in texts through the years.

In the field of measurement, perhaps the greatest change took place in the first 75 years included in this study. Technical material was omitted and measures similar in character but used for different purposes were combined, thus reducing the number of tables to be memorized. Finally, the extremes on the tables were dropped—those measures seldom used by the average person. Some of the modern texts have done a great deal of this type of simplification. In the *Child-Life Arithmetics* (1937), for example, the only information given in the table of liquid measure was:

> 2 pints (pt.) = 1 quart (qt.)
> 4 quarts = 1 gallon (gal.) (57d:291)

The bare essentials are included in such a table as this, but for the average person no further information is necessary. It is this

sifting out of unnecessary material that has simplified tables of measures found in textbooks today.

Simplification of vocabulary.—As would of course be expected, no attention was paid in the early arithmetics to the vocabulary used in the text. Common practice pointed the way, and almost every textbook writer followed the same path and used the same terms. Some terms had been handed down from the old days, when texts were written in Latin. The use of "facit," for example, is typical of such words. However, by the time the first books included in this study were written, most of the Latin had disappeared. The scholarly terms of the "learned" were still used, though, in a few texts. "Theorem" and "axiom" were sometimes found, and often long words like "promiscuous" and "compendious" were used. Colenso seemed even to try to make his text harder to understand, for he used abbreviations for mathematical terms, such as "Div^{d}" for dividend, "mult^{n}" for multiplication, and "rem^{r}" for remainder.

As has been said, Colburn was the first to write a text for children. In his attempt to simplify his arithmetic so that young people could understand it, he avoided some of the longer and more technical words. Other writers followed his example and it was not long before they were calling attention in their prefaces to the fact that their rules and definitions were stated in simple language, so that they could be understood more readily. However, authors used only their own personal judgment as to what could be understood by the school child. They apparently conducted no vocabulary studies, they had no standards to go by, and they made no word counts of the texts they developed. While the results of their efforts to simplify their content were evident, the simplification of sentence structure had more to do with the improvement at first than did changes in vocabulary.

Mention was found in the prefaces early in the twentieth century of the fact that pupils were having difficulties in interpreting the language of the arithmetics, but no specific reference was made to the scientific study of the vocabulary used. Thorndike's *Teacher's Word Book* (1921), a list of 10,000 (later increased to 20,000) words occurring most frequently in children's literature, the classics, newspapers, textbooks, correspondence, and other sources, set the standard for most texts published after that date. The first to mention that terms used in the text conformed to vocabulary studies was the *Modern Life Arithmetics* (1928). Most later texts made some mention of the fact that the words used in their books were carefully selected. The *Study Arithmetics* (1934-37) were the only ones to make an actual word count of each of their six books. Since word study has assumed such importance in recent years, it is believed texts in the future will use extreme care in selecting their vocabulary.

Division of material according to grade level.—The problem of fitting the text to the exact grade level needs of the child was not given serious consideration until comparatively recent times. Evidence of interest in arranging work according to grade level came

about 1821, when the first attempt was made to write a book for young children.

Later, interest in separating essential elementary information from the less necessary and more advanced phases of arithmetic led to the two-book series, which appeared first in somewhat disguised form and later became more or less standard for a period of 50 years (1870-1920). In the early series, the determination of material to be included in each text was more or less left up to the author who perhaps added an idea or two but in general followed the traditional form. In more recent years efforts in this direction have become much more scientific. The testing movement, which has grown by leaps and bounds in the last 30 years, has provided information believed to be fairly accurate concerning what may be expected of children of various age levels so far as their arithmetic learning is concerned. The results of scientifically conducted testing programs have been used by national committees to determine the grade level at which certain new topics should be introduced.

Studies of children's interests have also been made, and the later texts have been careful to provide problems and explanatory materials in a field that pupils of various stages of development will enjoy.

With the increased interest in grade leveling of texts has come a further breaking down of the arithmetic series, first into three separate books for the six grades, from 3 to 8, and, in the last three series studied, into six books, one for each grade. This has made it possible to set up rather definite limits of difficulty, vocabulary, and interests for each text, and thus to make the books as nearly scientific from these points of view as possible. Another advantage has been that the spiral system of teaching can be put to work effectively, as each successive grade can take up more advanced work in a phase of arithmetic that is an essential part of the subject through all the elementary grades.

Lesser Techniques

In almost every text were found suggestions or ideas as to interesting or useful ways of presenting material, but many related to only one principle or process and therefore could not be considered methods of sufficient importance to be discussed individually. It would be impossible, of course, to comment on all of them here—a few, however, were particularly interesting and will give an idea of the techniques used by different authors in presenting new subject matter.

In the early texts, and even as late as the present century, the majority of the suggestions of this type were directed to the teacher. In the first period, however, there was almost no mention of method, even in its minor considerations. Colburn went into a great deal of detail in his "key," which included explanations of each type of problem and suggestions as to means of presenting new material.

Colburn showed clearly the transition from the rule-book plan of teaching, with memorization as the aim, to the text designed to help the pupil understand his arithmetic. In discussing problems in multiplication, for example, he said:

> This section contains multiplication. The pupil will see no difference between this and addition. It is best that he should not at first, though it may be well to explain it to him after a while. (9:146)

And, in connection with division:

> This section contains division. The pupil will scarcely distinguish it from multiplication. It is not important that he should at first. (9:148)

Here he stressed the idea, dominant in early texts, that the textbook material is presented for the pupil to learn, whether he understands or not, but he did not leave the matter there. He felt that more than mere memorizing was necessary, and he indicated in these statements that an understanding was to be desired later on.

Explanation of symbols.—Educators were slow in taking up with the use of symbols, most of which were developed between the fifteenth and seventeenth centuries, because they felt symbols made written arithmetic less comprehendable and more abstract. When these were finally accepted, they were for many years given in a list in the front of the text as a sort of mathematical sign language to be learned before undertaking to study the text.

DeMorgan's (1832) was the first text included in this study to explain each symbol individually, as the process was introduced, but a few authors continued to list the symbols on a separate page until as late as 1878. As a rule, only a very brief statement was given with each, such as, "The sign + is used in addition."

Recent texts have elaborated on this and have been very successful in their efforts to make the meaning of each symbol perfectly clear. In the *Modern Life Arithmetics* (1928), for example, were such explanations as:

> Here is a new sign:—The sign − is called minus, and means less. It is the sign of subtraction. When you see the sign − between two numbers, you are to subtract the second number from the first number. Thus, 7−3=4 means 7 less 3 are 4. (54a:45)

One modification or another of this form has been found in most of the later texts, and is unquestionably a great improvement over the early "encyclopedia" form of explanation of symbols.

Different methods of performing the same process.—There has apparently been disagreement throughout the years covered by this study as to the best way to teach the fundamental processes. Some authors presented division as a series of subtractions, others as the reverse of multiplication, and others as a process totally unrelated to the other three fundamental processes.

In the older books authors sometimes suggested more than one way of performing an operation. Usually the purpose was merely to show how figures could be juggled rather than to teach the pupils another useful technique. A typical example of this was found in DeMorgan's text (1832):

First, it is evident that we may write the figures of any multiplier in a contrary order (for example, 4321 instead of 1234), provided that in the operation we move each line one place to the right instead of to the left, as in the following example:

2221	2221
1234	4321
8884	2221
6663	4442
4442	6663
2221	8884
2740714	2740714 (15:93)

The second method shown here is of no earthly use to a person if he knows the first method. In fact, it is more difficult to perform because it involves an extra step, that of reversing the multiplier. Therefore there seems to have been no excuse for including it except as a brain teaser.

Such material as this has been dropped from texts, but there still seems to be no agreement about the relative values of different ways of subtracting. First one method is in vogue, then another. Recently, authors have included more than one method, with the idea, perhaps, of giving the pupil a chance to use the one which is clearest to him. In subtraction there is little difference in the amount of work involved in the various methods. The *Iroquois Arithmetics* (1932) gave three methods of subtraction, as follows:

57	As 7 is less than 9, we change 7 to 17.
39	Think 9 and 8 are 17. Write 8.
——	We changed 7 to 17. Now change 3 to 4,
18	and think 4 and 1 are 5. Write 1.
57	As we cannot take 9 from 7, we change 7 to 17.
39	Think 9 from 17=8. Write 8. Add 1 to 3.
——	1+3=4. Think 4 from 5=1.
18	Write 1.
57	As we cannot take 9 from 7, we borrow 1 from 5.
39	This leaves 4. Change 7 to 17.
——	Think 9 from 17=8. Write 8.
18	Think 3 from 4=1. Write 1. (56a:48)

In the teaching of addition there has been similar disagreement, some authors telling the pupils to begin with the bottom number and add up, while others tell them to begin at the top and add down. However, the differences in amount of work involved and the chance for error in these methods and in those in subtraction are so slight that no one of them could be said to be much better than another. This fact probably accounts for the disagreement on the subject.

In division, however, there has been a more important controversy, that concerning the value of teaching short division as a separate process from long division. In all the arithmetics of the first 30-year period covered by this study, only one method of teaching division was included. However, short division then became popular and, after about 1850, only three texts failed to set up the two forms as separate

processes. One of these was a very recent text, the *Daily-Life Arithmetics* (1938). The arrangement of the figures on the page were not always the same in the various texts. Some authors placed the quotient above and some to the right of the dividend, and in short division some placed it below. French set his up quite differently, placing the divisor to the right of the dividend and the quotient immediately below the divisor.

Short cuts.—Short cuts were not popular in early textbooks, but were found in many more modern ones. In Peabody's text (1872) "Short Methods of Multiplication" (27:25) involved multiplying by multiples of ten and by such numbers as 13, 14, 31, and 41. The explanation of the short process, however, seemed to involve as many steps as the long process. In multiplying by 13, for example, the pupil was to multiply by 3 and by 10 and add the two together. This seemed to be more in the nature of an explanation of long multiplication than a short method.

Later suggestions as to short cuts were more practical. In the Smith series (1904), for example, the suggestion was made that in multiplying by aliquot parts "it is easier to multiply $1/8, $1/6, and $1/3, than it is to multiply 12½ ct., 16 2/3 ct., and 33 1/3 ct." (48b:67). And in the *Child-Life Arithmetics* (1936), in a section on multiplication, is a statement: "Remember: You may multiply by either number. It is easier to multiply by the smaller number" (57b:55).

Cancelling in fractions might be considered a short cut, too. In the beginning 30-year period of this study few books mentioned cancelling. Nicholas Pike suggested that equal numbers in numerator and denominator may be omitted, but Watson *et al.* (1805) were the only ones to use it in the present-day form and Talbott (1833) was the first to list it in the table of contents. Since that time it has been included in all the texts analyzed.

Poetry.—Some of the early texts stressed the use of rhyme in teaching rules and definitions. Many of these rhymes were so complicated and difficult to understand, because of their awkward word arrangement, that the pupil could not follow the thought through. By 1832 this method of teaching was dropped almost entirely, but one poem has lived through the years:

> Thirty days hath September,
> April, June, and November,
> All the rest have thirty-one,
> Save February alone;
> Which has but twenty-eight, in fine,
> Till leap-year gives it twenty-nine.

Many variations of this poem have appeared, but this version has been the most popular. The earliest was a puzzle form, in keeping with other content in the older arithmetics, beginning "The fourth, eleventh, ninth, and sixth, have thirty days to each affix'd" (12:37). With such a poem as this, memory work alone would be of little help, as the pupil would first have to figure out which the fourth, eleventh,

ninth, and sixth months would be. More recently, when the use of poetry was definitely out of style, a simplified form appeared, in which only the first two lines were quoted and the student was then told the rest of the information in prose. Apparently this method did not prove entirely satisfactory, for the entire six lines were again found in the latest texts.

Rules of proof.—In early texts rules of proof were as important as rules for solution of problems and accompanied them in almost every instance. Often rules of proof were also accompanied by examples. For instance, in the *Youth's Arithmetical Guide* (1805) the rules for the single Rule of Three and for proving a problem involving this rule were:

> RULE
>
> Multiply the second and third terms together, and divide the product by the first; the quotient will be the fourth term or answer, in the same name with the second.
>
> PROOF
>
> Invert the question, beginning with the answer, and the result will be the first term. Thus,
>
> If 5 yards cost 30 shillings, what will 15 yards cost?
>
> As *Yds.* 5 . . *s.* 30 : : *yds.* 15 : *s.* 30
>
> As *s.* 90 . . *yds.* 15 : : *yds.* 30
>
> 5)450 — 90)450(5 yds.
>
> 450 Proof.
>
> *Ans.* 90 *shillings.* (7:77)

Methods of proving answers are still stressed in texts but are given as suggestions or aids in checking answers rather than as additional rules to be memorized.

Use of blackboard.—The use of the blackboard in teaching arithmetic was stressed in many of the texts after the middle of the nineteenth century. Baker (1878) said "We cannot insist too strongly on the advantages of the blackboard exercises in developing the principles of science, and rendering them more easily and more thoroughly understood" (35:3).

Blackboard drills and games have been mentioned in this study in connection with drill work and motivation and therefore will not be discussed in detail here.

Use of analogies.—While the use of analogies is an effective way of helping a child to understand or to remember a principle, it was not often used in early texts. Two interesting examples were found however: the first, a footnote on the rule concerning multiplying length and breadth to find area, from Peabody's text (1872); and the second from the Hornbrook series (1898).

> Usage, and the want of any other equally terse form of language, sanctions this rule. Logically it is not true, any more than multiplying deer by grain will produce buckwheat. (27:110)

> Explain that interest is paid for the use of money just as horse hire is paid for the use of a horse, or rent for the use of a house. (45a:190)

Assistance in comprehension.—In early texts no help was given the pupils in understanding the principles involved in solving problems. Later, when reasoning became an aim in arithmetic teaching, a rather general suggestion was found in one or two texts, similar to the following from Peabody's (1872):

> Do not work for "the answer" so much as for the BEST WAY of finding the answer.
> Think out a PLAN of your work before you begin to work; if the problem be at all intricate, write out a formula. (27:151)

In recent years such suggestions and aids have become much more specific and consequently more helpful. The *Daily-Life Arithmetics* suggested four steps necessary in the solving of a problem:

> To be able to solve problems you must
> 1. Understand the problem.
> 2. Know how to add and how to subtract.
> 3. Know when to add and when to subtract.
> 4. Know the number facts. (58a:39)

The *Study Arithmetics* used this same idea, making it more personal in its attack:

> *Before you try to solve a problem, read it carefully. Then think of the answers to these questions before you do any figuring*:
> 1. *What does the problem ask me to find out?*
> 2. *Should I add, subtract, multiply, or divide?*
> 3. *What numbers should I use to find the answer?* (59d:20)

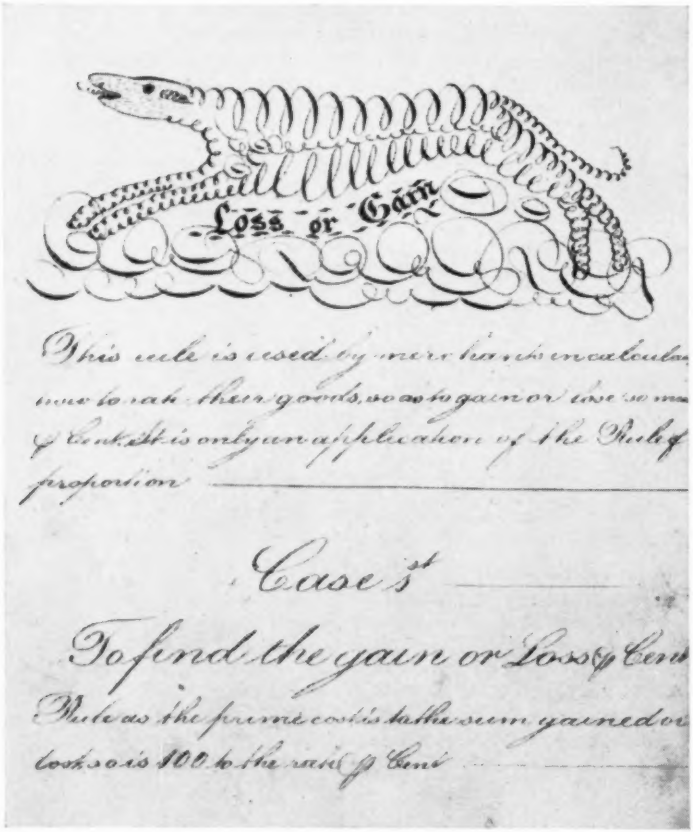
When the pupil's attention is called to the important factors in thinking through a problem, he is put more on his own resources and learns to meet new problems with greater understanding than when he has merely had each step in the solution set out for him almost as an entity within itself, with the total relationship not always made as clear as it should be.

Many other minor techniques could be mentioned, but a detailed discussion of them all would be almost endless. Those included here are representative, however, of minor methods used in the textbooks studied.

Motivation

Rivalry and desire for approval.—During the "ciphering-book" period, which was just drawing to a close when the first books analyzed in this study were published, pupils used sum books of their own making. These were usually covered with brown wrapping paper and contained eight pages of foolscap, folded together, on which the pupils copied sums "set" by the teacher. In some cases pupils who could afford it bought notebooks for the purpose. A great deal of care and effort was put into making an attractive sum book, and good pen-

manship was considered an important factor. (See Figure 13.) Emphasis was on accuracy and neatness rather than on comprehension, and success in learning arithmetic was judged by the number of pages of material copied carefully. The title of "Arithmeticker," a much sought-for honor, was earned by those who filled the most sum books with perfect copy, many of which contained very involved material. Rivalry for this



(one half size)

Fig. 13. Page taken from a sum book written in 1840

title created a form of motivation so satisfactory that no further need for methods of gaining the pupil's interest was felt. Seemingly, the aim of teachers and textbook writers was to provide material for those wanting to study the subject, rather than to create an interest in arithmetic, if their purposes can be judged by such statements as Hassler's:

> I intend to make a book . . . which merely a knowledge of reading, coupled with his [a lad's] own desire for instruction, would induce him to undertake as a study, both useful and agreeable; . . . . (11:ix)

When schools first began teaching arithmetic to young children, however, motivation became more of a problem. Colburn suggested mental arithmetic, as it lent itself well to competition, and this idea, which led into oral drill work, has been used as the basis for many kinds of motivation—games, contests, etc. Colburn also introduced the idea of leveling the content of textbooks to the abilities of the child, so as not to discourage him by confronting him with material "over his head." Simplification and clarification of texts have been important factors in attracting and holding the interest of the child, as has already been noted.

Still another idea promoted by Colburn, which was probably new in his day since children up to that time had ordinarily been taught individually rather than in classes, was that the teacher read a problem to the class, calling on one to answer. "In this manner every pupil will be obliged to perform the example, because they do not know who is to answer it" (9:143). This method of holding attention has been common ever since.

Through most of the nineteenth century and well into the twentieth, the method of motivation used most often was that of reward and punishment. The child who did his work well received 100% and was held up as a pupil to be patterned after, while the one who failed was disgraced in one way or another.

Creation of interest.—By the beginning of the present century the creation of interest was definitely taking the place of reward and punishment as a method of motivating children. One of the first moves in this direction was that of trying to make the appearance of the page less forbidding. Interest in the attractiveness of the text grew steadily—colored covers were used to catch the eye, pictures were added on both the cover and the inside of the book, and clever arrangement of material made the texts more appealing.

The subject matter was enlivened, too, by associating interesting things with the more routine mechanical phases of arithmetic. Games were suggested for teaching the number combinations, problems were "wrapped up" in more interesting content, and there was greater continuity of textual material.

Except for the fact that textbook writers of the last century have tried to present arithmetic as nearly as possible on the level of the child for whom the book was designed, for many years texts showed little real attempt to arouse or hold the interest of the pupils. Suggestions were sometimes made to teachers, either in footnotes or in special sections for teachers, as to methods that might be used for this purpose. Around 1900, these stressed particularly the value of providing real situations calling for the use of arithmetic and letting the children learn through experience. This was not new. It had been recommended by French in 1866 in the statement:

> You will add much interest to this study, by familiar conversations with the pupils about the pictures in the lessons, and the various objects represented in them. Many of the pictures represent mechanical, manufacturing, and other business operations and industrial pursuits, about which children should be instructed. Encourage them to visit factories, shops, and other places of business of the kinds represented in the pictures, or suggested by them or by the examples, for the purpose of obtaining information. (29a:108)

While this plan later became very popular, it was not until long after French published his text. French was the first to make a sincere attempt to relate arithmetic to life outside of school. Out of this effort grew the social arithmetic of today as well as the inclusion of such material as games and stories.

The blackboard, when it first came into general use, was considered a great motivating force. Textbooks suggested blackboard drills that could be used by the pupils—a form of game in which all could partake. Baker's *Elementary Arithmetic* (1878), for example, stated that *"the Author cannot too highly recommend to the Teacher the use of the Blackboard described on the following page"* (35:5). The page referred to contained two columns of numbers, the one on the left-hand side of the page containing numbers from 1 to 36 and the one on the right-hand side from 37 to 72. It was suggested that columns like these be painted on the margins of the blackboard and that pupils perform their drill work by the use of slips with which to cover certain numbers or by check marks made in chalk. This could be carried on in the form of a game, similar to a spelling bee. Other texts contained games in which three rows of three numbers each were drawn on the blackboard. The center number was the one usually used to add to, subtract from, multiply by, or divide by each of the others. The teacher, or a pupil from the class, pointed to the numbers to be used in the combinations and the others gave the answers.

Another game, played without the use of the blackboard, was suggested in the Hornbrook series (1898):

> Playing "Numbers Out," a device for learning the multiplication table, is contributed by a very successful teacher and warmly indorsed by her pupils. In Numbers Out, the children stand around the room, leaving one side of the room where there is a blackboard vacant. Beginning at one end of the class, they number themselves 1, 2, etc. In playing "Threes Out," when 3 is reached, or any multiple of 3, the child, instead of calling the number, says "Out," goes to the blackboard, writes his number large and bold as high as he conveniently can, and takes his stand under it. When a sufficient number of children are out, the teacher calls on them to make statements about their numbers. "I stand for 27 or 9 threes," says one. "18 is my number. It equals 6 threes," says another. A child who in numbering around names a multiple of 3, or who says "Out" for any number that is not a multiple of 3, or who makes a wrong statement about his number, misses the game. (45a:132-3)

Games in recent texts are often included as sections of the text, like "Tossing the Ring" (57a:10) in the *Child-Life Arithmetics*, a game in which the teacher is to draw squares on the floor with a

number in each and the pupils toss a ring onto a square. Each child keeps a record of the squares he hits with the ring and adds his own score. This game idea had been used a little earlier by the *Triangle Arithmetics* in a target game in which the children aimed at numbers on a target and added their scores.

Dramatization was used in a few series, especially in the *School Arithmetics* (1919), to create interest. Early in the elementary text the authors gave a picture of four children on a donkey, followed by questions in fine print at the bottom. Their comment on this material was that:

> These problems show what is called the dramatization of arithmetic; that is, acting out a real situation. Such problems are more real when planned and suggested by the teacher than when they are given in a textbook. (52a:2)

On the following page were further suggestions along this line in another footnote:

> Among the common rimes that can be dramatized at this time are the following: "Bo-Peep," "Going to St. Ives," "Old Mother Hubbard," "Ten Little Indians," "Three Little Kittens," and "The Old Woman Who Lives in a Shoe." Among the common stories are "Jack and the Beanstalk," "The Three Bears," and "Jack the Giant Killer." Among the dramatized occupations are making a garden and running a grocery store. (52a:3)

Development of a feeling of need for arithmetic.—Perhaps the most recently introduced form of motivation, and one which has been approved and used by practically every modern textbook writer, is that of creating in the child a realization of his need for number in his everyday life. The theory is that if a child can be made to *want* to learn his arithmetic, half the battle is won. To develop this desire, writers have been showing the pupil the correlation between his arithmetic and his experiences both inside and outside of school. The stories included in some of the texts have proved valuable in this respect. Geography provides a background for problems concerning travel, mail, telegraph, etc.; civics has been used in problems concerning public debt, interest, bonds, and so forth; and, outside of school, keeping scores in baseball and other games, buying tickets for shows, getting correct change, doubling cooking recipes, measuring heights and weights of children, and many other activities have been found valuable in teaching the significance of arithmetic. Subjects of this sort are not new, but in the past they were used merely as different forms of "clothing" for problems, not as motivating factors with real life interest.

The social uses of arithmetic have been made more and more prominent in texts in the last few years. The *Study Arithmetics* (1934-37) have done exceptionally well in making the child think about the mathematical relationships involved in carrying on business, owning a home, financing a community project, or arranging intercommunity contacts. For example, in their eighth grade text, the following questions about the merchant's relationship to the community were found:

1. Why are most stores in a community usually found in one group known as the business district?

2. Why are merchants willing to pay higher rents for store buildings on main streets than for those on side streets?

3. In what ways does the outside appearance of a store affect a merchant's business?

4. Why are the streets usually better lighted at night in the business district than in the residence districts?

5. Why is a clothing merchant often willing to sell overcoats for $20 in February exactly like those for which he received $30 in November?

6. What are some other articles that can be bought more cheaply at one season of the year than at another?

7. Why is it sometimes foolish to try to save money by waiting to buy such goods until they are cheaper?

8. In many grocery stores the clerks use an adding machine to find how much to charge each customer for his order. In what way may this help the customer?

9. Why is an adding machine more useful to a clerk in a grocery store than to a clerk in a shoe store?

10. Why is a telephone a necessity for a merchant?

11. Why does the telephone company usually charge a higher rate for a phone in a store than for a phone in a private residence?

12. Why do both merchants and their customers use per cents and decimal fractions a great deal more than common fractions in buying and selling goods? (59f:114)

These problems do not concern number as such, but they do show the relationship of number to the ordinary affairs of life. Thus, in the last few arithmetics included in this study, authors have attempted to make the child "live" the subject, to feel it is a part of his life, and a useful and interesting part, just as his home, his friends, and his daily experiences are. He is made to realize that in almost everything he does there is some need for arithmetic and that life is more meaningful and more understandable to one who can grasp the numerical relationships of the things that take place about him. This realization is a motivating force, for it creates within the child a feeling of need for information and a desire to learn more about the subject.

Chapter VI
Written Problems

WRITTEN problems probably show the changes that have taken place in the teaching of arithmetic more clearly than does any other one factor. The processes taught in the text, the practicality of the content as judged by the subject matter of problems, the simplification of texts as seen in the clarification of statement and the use of smaller numbers, the grade leveling of material as shown in the appeal to the interests of children of certain ages, the tendency to correlate arithmetic with interesting subjects outside the field, and many other phases and trends in arithmetics are evident in the content of written problems.

Written Problems Common to All Textbooks

Since the content of arithmetic texts has not changed a great deal in the 150 years covered by this study, there has been no radical difference in the topics with which the written problems have dealt. In general, it may be said that the problems fall into four main classifications—practical business problems, problems involving measurement, problems based on historical or scientific facts, and problems concerning the experiences of children in school, in play, or in other daily-life activities. The terminology of the problems has of course changed greatly, partly because the American vocabulary has changed and partly because authors today are making a distinct effort to keep the wording on the level of the child. The use of numbers in problems principally for the sake of giving the student experience with handling different figures is no longer accepted as good practice; today numbers are used that are plausible and, if possible, absolutely correct and in keeping with the subject matter of the problems. The point of view has also changed from that of including only problems on business matters to that of using subject matter with which the child is acquainted, either in his own personal experiences or in his family's activities. These are graded to his changing needs and interests and kept always within his range of experience.

Practical business problems.—Most of the problems in arithmetic texts have been based on business practices of one kind or another, for, until very recently, the principal goal of arithmetic teaching has been to train students to handle various business dealings efficiently.

Beginning with the very earliest of the textbooks, business arithmetic has been considered of vital importance, and many authors have claimed that their texts were especially practical from this point of view. The inclusion of sections on bookkeeping, the teaching of United States currency, and, much later, the introduction of practical problems in various vocations have all been evidences of the attempt to meet practical needs.

By 1875 authors were becoming somewhat more specific in their statements concerning the practicality of their problems. Raub (1877)

claimed to include problems from practical business and life; the Milne series (1892-93) stated that it presented business methods rather than school methods; the Indiana Schoolbook series (1897-98) included an entire chapter on occupations, with sections concerning the lumber dealer, the farmer, the carpet dealer, and so forth; the *Modern Arithmetics* (1913) stated that their problems had a "vocational character"; and almost all the more recent texts have advertised their practicality in one way or another.

Throughout these texts, business problems have included such subjects as interest, commission, insurance, discount, profit and loss, bills and accounts, etc. While the terminology of the old problems was very different from that found in texts today, the content, as shown in the following, was much the same.

> Cast up the Interest of *l*.886 16 0, for 1 Year, 11½ Months, at 4 *per Cent, per Annum?* (4:215)
>
> A House being let upon a Lease of seven Years, at 50*l*. *per Annum*, and the Rent being in Arrear for the whole Term, I demand the Sum due at the end of the Term, Simple Interest being allowed at 4 *per Cent per Annum?* (4:227)
>
> What come the commissions of £785 to at 3½ guineas per cent? (3:132)
>
> A Merchant adventured 500*l*. from Boston to Philadelphia, at 3 per cent. from thence to Guadaloupe, at 4; from thence to Nantz, at 5; and from thence, home, at 6 per cent.; What will be the premium, tantamount to those given in the question, on a policy for covering the first adventure, the voyage, supposing the risques out and home equal? (3:294)
>
> Some Merchandizes being bought for 5150*l*. with 4½ Months Discount, at 8 *per* Cent *per Annum;* besides 1 *per Cent* for prompt Payment. How much ready Money must I pay? (4:230)
>
> What are the avails of Charles L. Fargo's note, discounted at 10 per cent per annum? (27:225)
>
> A draper bought 100 yards of broad cloth for $56; I demand how he must sell it per yard, to gain $15 in laying out $100? (16:131)
>
> Sold sugar at 9 cents 6 mills, by which I lose 25 per cent. what was the prime cost? (14:117)
>
> Having a mind to buy a suit of new clothes, I went to the tailor's, to see how much money would be necessary for that purpose; when I found he would charge for a coat 30 dollars, for a pair of pantaloons 12 dollars, and for a waistcoat 5 dollars: what will the suit cost at that rate? (12:14)
>
> Expended at one 225 D. 75 cents; at another . . . , and lastly, 27 D. 22½ cents; how much was expended in all? (7:29)
>
> A gentleman left his son 725 dollars more than his daughter, whose fortune was 15 thousand, 15 hundred and 15 dollars; what was the amount of the whole estate? (10:28)
>
> Francis has 35 head of cattle on his farm, and his neighbor James 84; how much has the one more than the other? (11:100)

Answers were included after almost all these problems, and also, sometimes, a short explanation of the method of finding the answer. In this last problem, for example, the author showed how to work the

problem, but he became somewhat involved in words at the end when he attempted to label the answer:

James's cattle	84
Francis' cattle	35
Difference	49 which James has more. (11:100)

As can be seen from these examples, many of the problems were nothing more than abstract problems; that is, they simply asked for a certain process to be performed. There was no story connected with them and no context other than the information essential for carrying on a certain mathematical performance. Even when authors attempted to clothe the figures in some sort of a story, the problems remained strictly impersonal. "A merchant," "a lady," or some other indefinite person was usually the agent in the problem. The subjects of the problems, too, were broad and general—"a parcel of goods," "some merchandizes," or some other all-inclusive term was often used, probably because authors wished to cover as much territory as possible since their texts were designed for persons in various kinds of business.

In sharp contrast to this is the form used in stating problems in modern texts. In these there is not only a story involved in the problem, but often a story runs through a series of problems, unifying their content. Persons involved are given names, so that they seem real; the activities they carry on in the business world are specific, so that the child can visualize the situation clearly; and the experiences discussed are within the realm of the child's knowledge. The following are examples of problems from recent texts which cover the same principles as are tested in the problems already quoted:

> Mr. Laird borrowed $4000 to buy a home. He promised to pay 6½% interest a year. How much interest is that? (58d:264)
>
> Ted had $25.38 in his savings account. The bank does not pay interest on parts of dollars. What would be his interest at 2% for a year? (58d:263)
>
> A person insured his life for $7000 at an annual premium of $36.40 per $1000. After making 7 payments he died. How much more did his heirs receive than he had paid? (49b:183)
>
> An agent sells our school $85 worth of maps, and his firm pays him 7½% commission. How much is this commission? (52b:181)
>
> Mr. Smith bought a suit at $36.00 in July. What would he have saved if he had waited for the August sale and bought at 33 1/3% off? (56b:311)
>
> Dresses that had been marked $6.95 were reduced to sell for $5.49. How much were these dresses marked down for the sale? (57c:18)

Problems like these are familiar to all today. They deal with things known to children and they are often personal enough to be interesting to the pupil. Many new ideas in wording problems or giving them meaning beyond that of the arithmetical processes in-

volved have been introduced in new arithmetics, but these will be discussed more in detail later in this chapter.

Problems involving measurement.—Since measurement is one of the most common and most practical uses for a knowledge of number and is one which children often meet in their everyday life, it has always been a subject for arithmetic problems. In the older texts, however, the problems were not kept within the realm of the child's experience. They often involved very unusual measurements for the sake of testing the pupil's acquaintance with the tables and his ability to manipulate large or complicated figures. Some of the older problems in measurement were:

> If a pistareen be worth 14 2/5 pence, what are 100 pistareens worth? (8:167)
>
> In 24796800 seconds, how many weeks? (8:72)
>
> How many yards of baize which is 1¼ yards wide, will line 18⅞ yards of camblet ¾ yd. wide? (8:168)
>
> Subtract 95 yards, 3 quarters, 2 nails, from 156 yards, 2 quarters, 3 nails. (6:51)
>
> Change 794 guilders, 15 stivers, current money, into bank florins, agio 4⅜ *per cent.* (2:87)
>
> How many bricks 9 inches long and 4 inches wide, will pave a yard that is 20 feet square? (6:196)
>
> How many barley-corns will reach round the globe, it being 360 degrees? (8:71)
>
> How many silver thimbles, each weighing 1 pwt. 3 grs., may be made from a silver dollar? (27:142)

The weight of various United States coins was an important item and was given in most textbooks. Although many problems were based on this information and on the various rates of exchange, there was for years no standard way of writing dollars and cents. Problems appeared in many forms, among them the following:

> What is the monthly interest of 9D. 59c. 7m. at 6 per cent. per annum? (5:272)
>
> A vintner has wine at 130 cents, at 160 cents, and 180 cents per gallon; and he would have 32 gallons worth 145 cents per gallon: I demand how much of each sort he must have? (12:156)
>
> There is an excellent well built ship just returned from the Indies. The ship only is valued at 12145 dols. 86 cents; and one quarter of her cargo is worth 25411 dols. 65 cents. Pray what is the value of the whole ship and cargo? (8:24)

The answer to this last question was given as "$113792, 46 cts."

Many questions on the reduction of "Federal Money to the currencies of the several United States" (8:93) were similar to:

> Reduce 120*l.* 10*s.* Massachusetts currency, into South-Carolina and Georgia currency. (8:99)

At the time, these problems involving exchange were very practical, for a knowledge of rates of exchange and how to use them was needed daily in carrying on business transactions. Today, of course,

there is little need for such problems. Tables of currencies have been omitted from the more recent texts, and the study of measurements has been limited to those measures used in everyday situations. Problems based on the tables of measures are, from the modern point of view at least, much more practical than they formerly were.

> Tom heard his father say to a neighbor, "My wheat averaged 32 bu. 3 pk. to the acre, and I had 50 acres in wheat." How many bushels of wheat did Tom's father raise that year? (54b:249)
>
> Milk costs 5¢ for a half-pint bottle or 12¢ for a quart. Which is cheaper? How much cheaper is it? (58c:136)

On the whole, problems in measurement have been brought more nearly within the compass of the child's experience. Many of them have correlated subject matter from other fields with that in arithmetic, thus adding interest to the problems as well as increasing the pupil's range of information.

Problems based on historical or scientific facts.—While it might be said that every arithmetic based at least a few of its problems on historical or scientific facts, the information used in some of the early texts could not be relied upon. It seems, therefore, that, while the idea of using history or science as background for problems was not new at the time the earliest arithmetics in this study were published, it was many years before authors considered it necessary to use accurate figures. Pupils are taught today to use what they learn in one class when they find need for it in another class, and they probably would not be particularly happy to find that the figures in their arithmetic problems gave them incorrect information for their history or geography class.

Examples from early texts of problems supposedly based on fact are:

> Rome being built 817 years after the Pyramids of Egypt, and 146 years before the Christian Era, America having been discovered 1594, the Declaration of Independence by the United States having taken place 182 years afterwards, and the year of it now counted being 52; how long is it since the building of the Egyptian Pyramids? (11:99)
>
> The diameters of the principal planets are as follows: Mercury 16057000 feet, Venus 40125400 feet, the Earth 41836420 feet, Mars 21646200 feet, Vesta 212000 feet, Juno 12389460 feet, Ceres 2125480 feet, Pallas 27619700 feet, Jupiter 4643343000 feet, Saturn 41763300 feet, Uranus 181210000 feet; how much would they make in distance supposing them all placed together? (11:99)
>
> An analysis of the Board of Trade returns for 1861, respecting shipwrecked lives, gave the following results:—Saved by life-boats, 13½ per cent.; by rocket and mortar apparatus, 8 per cent.; by ships' boats, &c. 6 per cent.; by individual exertion, ½ per cent.; lost, 16 per cent. Determine the number of lives saved, by the several means enumerated, corresponding to an excess of 2619 rescues by ships' boats over those by life-boats. (23:184)

The Board of Trade figures in this last problem may have been correct, but the chances are that they were used as filler to make a

more concrete situation for the problem and that they were not necessarily accurate. The fact that the discovery of America was given as 1594 makes one wonder about other figures. The diameters of the planets are, of course, ordinarily given in miles rather than feet, but feet were probably used in order to make a more complicated problem in addition. However, there is an error in the figures, for Jupiter is given as about ten times too large while Saturn is ten times too small. Also, present-day astronomers must differ from old-time ones as to the principal planets, for Vesta, Ceres, Juno, and Pallas are not classed among the principal ones today. Since Neptune and Pluto had not yet been discovered at the time Hassler's text was published, they of course were not included.

More recent authors have been more careful about the information they give in problems. They not only attempt to be accurate, but also to include facts that the children should know. The *Daily-Life Arithmetics* (1938) seemed to have succeeded especially well in providing questions of this type, all of the following being taken from that series.

> Sound on a radio travels approximately as fast as the speed of light. During a storm the static from a bolt of lightning was heard on the radio 5 sec. before the clap of thunder was heard. The temperature was 86°. How far away was the lightning? (58f:157)
>
> Remember that in most places water boils at 212°. How much hotter is boiling water than ice? (58b:134)
>
> Remains of very large animals have also been found. Scientists have called one such animal the dinosaur (pronounced dī'nō-sôr). The dinosaur is said to have reached a size of 70 ft. in length. If a dinosaur of that length tried to come into your schoolroom, how much of him would have to stay outside? (58d:31)
>
> Sports writers sometimes call the 220-yard dash the "furlong." This is an old unit that meant "a furrow long"—the distance a farmer would plow with his oxen before stopping to let them rest. How many furlongs are there in a mile (1760 yd.)? (58e:230)

These problems are designed to reach children whose interests lie in different fields—in science, history, athletics, etc. Almost every field of knowledge is touched upon somewhere in the problems included in this series as well as in many of the other modern texts.

Problems concerning children's daily-life activities.—By daily-life activities, here, are meant the activities which the child sees as a part of his daily life. Some of them concern him in his personal experiences, when he attends school, when he plays games with other children, when he earns a little money doing odd jobs, and so forth. Others concern the activities which he sees take place about him; that is, the paying of utility bills, the buying of groceries, the sending of mail, etc. Although much has been said about the fact that modern texts are emphasizing these activities in their problems, it must not be assumed that such things were not mentioned in early texts. Very few problems of this type were found in the first period of this study, but they became more common after the publication of

Colburn's text, which brought the child rather than the subject into the foreground. Two problems of this kind from old arithmetics are:

> A gentleman coming into a school where the boys sat remarkably quiet, gave to each 6¼ cents; the money he gave amounted to $2 75 cts. how many boys were there? (14:173)
>
> A boy, having a pound of almonds, said he intended to give ⅓ of them to his sister, and ¼ of them to his brother, and the rest to his mamma. His mamma, smiling, said she did not think he could divide them so. O yes I can, said he; I will first divide them into twelve equal parts, and then I can divide them well enough. Pray how many twelfths did he give to each? (9:113)

Few problems, even in the most recent arithmetics, are more personal than this last problem, which was found in Colburn's text in 1821. The wording, of course, is a little stilted, but in all probability it did not seem so much so in those days. Problems found in some of the later texts include:

> Lucille is making doll dresses to sell for Christmas. How many dresses can she make out of 2 yd. 1 ft. of material if it takes 8 inches for each dress? How much will she make if she sells the dresses for 25¢ apiece and the material cost her 30¢ a yard? (57e:68)
>
> A baking oven consumes 30 cubic feet of gas an hour. It requires 2 hours to roast a chicken. What is the cost of the gas consumed at $1.30 a thousand cubic feet? (55c:161)
>
> Marie's breakfast was a banana and ⅝ cup of oatmeal with a tablespoon of sugar and ½ cup of milk. How much more or less than enough calories was this for Marie, who is 12 yr. old? (58e:250)
>
> A boy takes 9 subscriptions to the "Youth's Companion" at $1.75 each. How much money does he receive for them all? (46a:202)
>
> On Monday the sixth grade had a spelling test of 25 words. The teacher marked the papers and wrote some of the marks on the board. Helen got 64%; John, 92%; Margaret, 96%; Arthur, 60%; Henry, 84%; Jean, 80%; and James, 72%. How many words did each one miss? (54b:320)

When confronted with problems of this type, the child can see the everyday use of arithmetic.

Throughout the textbooks of this study there has been evident a tendency to make the written problems more personal and to give them meaning and life. The trend toward simplification has perhaps been the most important factor in changing the way in which the problems were presented, while the attempt to make arithmetic practical from a daily-life point of view has done much in changing the content.

Problems Typical of Old Arithmetics

Certain types of problems found in old texts are seldom if ever found in more modern ones. This may be because topics have been dropped from arithmetics, because simplification in wording and thought have made the old form undesirable, because rearrangement in subject matter has shifted the topic to higher mathematics, or because of some other factor. (See Figure 14.)

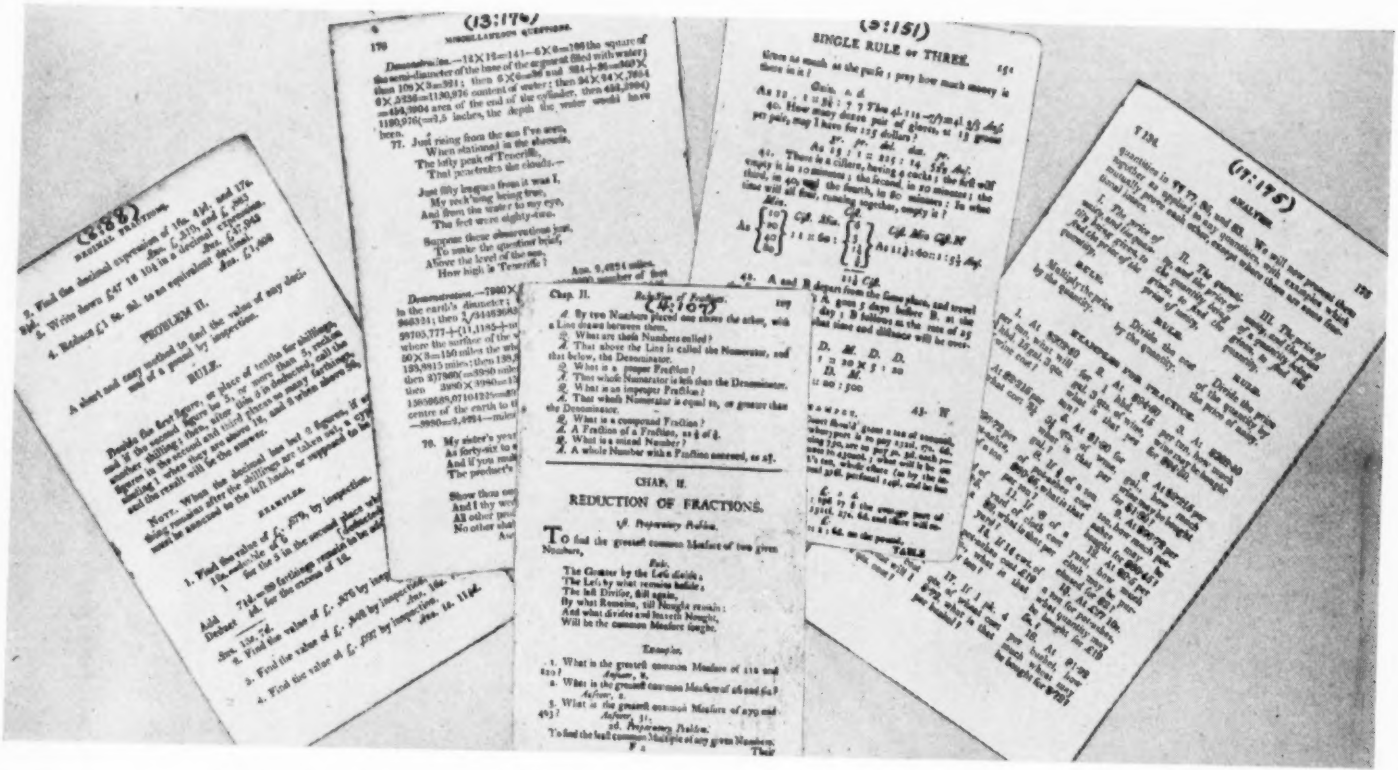


Fig. 14. Typical pages of written problems found in arithmetic textbooks published before 1850 (approximately one half size)

Questions on rules and definitions.—A knowledge of and ability to quote rules and definitions were considered vital to the study of arithmetic in early days. Often texts gave little drills following the rules and definitions at the beginning of a new process or principle. Since all old texts included answers with their questions, these drill questions were followed immediately by the answers. Sometimes the answers to these were complicated, but Gough (1801) thought putting them in rhyme might make them more easily remembered. Throughout his text here and there were found such questions and answers as:

Q. How must I reduce a Fraction to its least Terms?
A. By the Measure, the greatest of the Numerator,
Which likewise will measure the Denominator;
Divide both the Terms of the Fraction; 'twill find
The Terms of a Fraction the least in their Kind.
But rather than thus it may probably please
The least Numbers to approximate by Degrees:
If the Numbers be even still 2 will divide,
But an odd Number always in odd must be try'd;
If Cyphers end,—both of like Cyphers deprive;
5, or 5 and a Cypher?—divide them by 5. (4:117)

For many years the idea of questions and answers on rules and definitions was very popular, in spite of the fact that the answer was there to read immediately following the question. It is possible that teachers read these questions to the pupils, who had their books closed, and that the teachers needed the answers at hand, but no mention of this method was found in the texts. Smiley explained that "the Interrogations which follow the Rules, are intended as an exercise to direct the attention of the learner in a particular manner to the Rules, and to fix them more permanently in the memory" (12:vi). Since the study of arithmetic at that time was almost entirely copy work and memorization, questions set up in this form were not as undesirable as they at first appear.

Later, more informational material was included in texts, but this same catechism type of questioning was used in testing the student's understanding of the material presented. As late as 1852, in Davis' *First Lessons in Arithmetic*, questions of this kind based on a table of Federal money were found.

1. What is the currency of the United States?
Federal money is the currency of the United States.
2. What are its denominations?
Its denominations, or names, are eagles, dollars, dimes, cents, and mills.
3. Of what are the coins of the United States made?
The coins of the United States are of gold, silver, and copper.
4. Which are gold?
The eagle, half-eagle, quarter-eagle, and dollar.
5. Which are silver?
The dollar, half-dollar, quarter-dollar, dime, and half-dime, and three-cent pieces.
6. Which are copper?
The cent and half-cent. (20a:87-8)

Such information as this was not mere background material. Many of the problems concerned the amounts of silver or copper in certain coins, the relative weights, and so forth—unnecessary facts which probably are not known today by 99 per cent of the population. Questions about rules and definitions continued to be found in some of the texts even up to the late nineteenth century, but answers did not accompany them.

Questions on a certain topic.—In the first texts of this study, as has been said, each new rule (and there was a new one on almost every page) was followed by problems to be solved by that rule. Usually from 5 to 15 problems were included. These were often headed "Examples" or "Application" instead of "Problems" because they were included for the purpose of showing the student where the rule applied and how it operated. Often the solutions were given for at least the first two or three of these examples. They were worked out in detail and included the proof. A definite attempt was made, however, to vary the subject matter of the problems as well as the numerical values.

Walkingame's text was typical in this respect. His discussion of alligation alternate began with the heading, followed by the definition. Then came, in order, the set-up of the problem (the two terms of the extremes and the two of the means being bracketed and called "yokefellows"), the rule, and the proof. Lastly, were six problems based on the rule, the first one of which was worked out in detail:

> 1. A vintner would mix four sorts of wine together, of 18*d*. 20*d*. 24*d*. and 28*d*. *per* quart—what quantity of each must he take to sell the mixture at 22*d*. *per* quart?
>
> 2. A grocer would mix sugar at 4*d*. 6*d*. and 10*d*. *per* lb. so as to sell the compound for 8*d*. *per* lb.—what quantity of each must he take?
>
> 3. I desire to know how much tea, at 16*s*. 14*s*. 9*s*. and 8*s*. *per* lb. will compose a mixture worth 10*s*. *per* lb.?
>
> 4. A farmer would mix as much barley at 3*s*. 6*d*. *per* bushel, rye at 4*s*. *per* bushel, and oats at 2*s*. *per* bushel, as to make a mixture worth 2*s*. 6*d*. *per* bushel—how much is that of each sort?
>
> 5. A grocer would mix raisins of the sun at 7*d*. *per* lb. with Malagas at 6*d*. and Smyrnas at 4*d*. *per* lb.—I desire to know what quantity of each sort he must take to sell them at 5*d*. *per* lb.?
>
> 6. A tobacconist would mix tobacco of 2*s*. 18*d*. and 15*d*. *per* lb. so as the compound may bear a price of 1*s*. 8*d*. *per* lb. —what quantity of each sort must he take? (2:76-7)

These were the only problems on the subject, the whole topic having taken up less than a page and a half. "Alternation Partial" then followed, with about the same amount of space allotted to it, and then came "Alternation Total." These were, of course, variations of the same general rule, but nowhere in the text was their relation made evident. In some arithmetics the topics were never mentioned again, but in the back of Walkingame's text 122 problems on miscellaneous topics were listed, only three or four of which, however, dealt with alligation.

Problems in verse.—As might be surmised from the question and answer concerning the reduction of fractions to lowest terms, "poetry" was a favorite form in old texts, particularly in those of Guthrie (1810) and Ostrander (1829). Problems in verse ran all the way from a simple question, like:

> Just fifteen pair of ladies' gloves,
> For sixty dimes had I;
> How many pair of that same kind
> Will forty eagles buy? (13:166)

and

> One hundred dollars you divide
> Between two worthy men,
> That when each part is fairly squar'd,
> The difference is but ten. (13:168)

to long involved ones, like:

> I happened one evening with a tinker to sit,
> (Whose tongue run a great deal too fast for his wit,)
> He talked of his art with abundance of mettle;
> Then I asked him to make me a flat-bottom'd kettle,
> The top and the bottom diameters to be
> In just such proportion as five is to three;
> Twelve inches the depth (I proposed) and no more,
> And to hold in ale gallons seven less than a score.
> He promised to do it and to the work went?
> But when he had done it he found it too scant;
> He altered it then, but too big he has made it,
> For though it holds right the diameters fail it:—
> Thus by often making it too big and too little;
> The poor tinker at last quite spoil'd his kettle;
> But he said he would bring his said purpose to pass,
> Or else he would spoil every ounce of his brass.
> Now to keep him from ruin, I pray find him out
> The diameter's length—for he'd ne'er do it I doubt?
> (14:199)

Some of the rhymes were particularly interesting from the point of view of both form and content. Some were very complicated and very poorly stated, while others were fairly clear. One of Ostrander's that was stated clearly, but involved a rather complicated process, was:

> I plac'd a bowl into the storm,
> To catch the drops of rain—
> A half a globe was just its form,
> Two feet across the same;
> The storm was o'er, the tempest past,
> I to the bowl repaired—
> Six inches deep the water stood,
> It being measur'd fair;
> Suppose a cylinder, whose base
> Two feet across within,
> Had stood exactly in that place,
> What would the depth have been? (13:175)

In textbooks of the last half century the subject of drinking and intoxication has been carefully avoided. However, it was a common subject in the early days. One of the problems based on drinking was:

A, B, and C, met at an inn,
Where they grew very merry
With drinking brandy, wine, and gin,
Some glasses too of cherry;
At length the reck'ning they must pay,
To make all matters even—
A paid two ninths, as poets say,
While B and C paid seven;
As oft as B paid eighty cents,
C paid the sum of fifty;
Now tell the cents each fellow spent,
If C spent nine times twenty? (13:167)

This problem bordered very closely on the puzzle type, as did many of those written in verse. The answer given in Ostrander's text was "A paid 1,33 5/7 cts.; B, 2,88 cts.; and C, 1,80 cts." It is evident from the answer that the problem was written purely for the sake of giving the student figures to manipulate, rather than as a practical problem, for cents cannot be divided into sevenths.

One of the favorites was found in Guthrie's text:

A landed man two daughters had,
And both were very fair;
To each he gave a piece of land,
One round, the other square;
Twenty shillings to an acre,
Each piece its value had;
The money which did compass each,
For it exactly paid.
If 'cross a shilling be an inch,
(As is it very near)
Which was the better fortune? she
That had the round or square?
The number also is required
Of acres each one own'd
And now, sir, you are desired
To make those numbers known? (14:198)

This problem appeared in different texts in slightly varying forms. Sometimes it was a gentleman or a merchant instead of a landed man, and he used dollars instead of shillings for measurement, but the principle was exactly the same.

Bordering on the puzzle type (although today any problem in verse would probably be considered a puzzle problem) was one concerning the number of days an artist worked for a man.

A prudent man for 30 days did hire
An artist, who for wages did require
Seven shillings for each day he wrought,
But nothing when full idle he was caught;
But to each of those days to his master he'd give
Five shillings, like money, (so together they'd live)
Thirty days being pass'd, they came to account,
And found all the sevens (when added) amount
Exact to the sum which the fives did produce,
For days when the artist was idle and loose:—
Now, no money to master or man being paid,
Say when the man wrought, and how often he play'd;
(14:175)

Problems in rhyme were almost never found in textbooks after about 1835. This was probably because of the trend toward simplification, although the fact that arithmeticians usually are not poets may have had something to do with it, too. The tendency to include puzzle problems continued to develop, however, but they were stated in prose.

Puzzle problems.—Through the first half of the period covered in this study, puzzle problems were very popular. Ability to manipulate figures was an art to be desired, and puzzle problems were believed to develop this ability. Some were purely catch questions—the kind found in contests today or in tests of alertness and clear thinking, like:

> What is the difference between six dozen dozen, and half a dozen dozen? (10:49)
>
> A snail in getting up a May-pole only 20 feet high, was observed to climb 8 feet every day, but every night he came down again 4 feet—in what time, by this method, did he reach the top of the pole? (2:172)

Probably most people today are better acquainted with the frog trying to jump out of the well than with the snail climbing the May-pole, but the principle is the same.

As a rule, the puzzle problem seems to have been designed to check on a pupil's ability to analyze a problem and to follow through the various steps without getting lost on the way. It fitted in nicely with the theory of mental discipline, for a pupil able to keep the details of a complicated problem in mind and to "stick with" the problem to the finish had to have a strength of mind and will above the average. A few examples give an idea of the complications that could be met with in some of the problems in early texts:

> Suppose that in a meadow of 20 acres the grass grows at a uniform rate, and that 133 oxen could consume the whole of the grass in 13 days, or that 28 of the oxen could eat up 5 acres of it in 16 days; how many of the oxen could eat up 4 acres of it in 14 days? (23:174)
>
> There is a certain number, which being divided by 7, the quotient resulting multiplied by 3, that product divided by 5, from the quotient 20 being subtracted, and 30 added to the remainder, the half sum shall make 65; can you tell me the number? (8:212)
>
> A man, driving his geese to market, was met by another, who said, Good morrow, Master, with your hundred geese; says he, I have not a hundred; but if I had half as many more as I now have, and two geese and a half, I should have a hundred; how many had he? (9:139)
>
> Divide the cube root of $\frac{2460375}{4096}$ by the fourth root of 50625. (23:133)
>
> If a ton of turnips will last 25 sheep for a fortnight, how much will be required to supply 40 sheep during the months of January and February in Leap year? (23:98)
>
> In a certain manufactory, 158 men of ordinary ability and working the same number of hours each day, execute a certain piece of work in a week; but if the abilities of 2 of them had been, respectively, 1/7 and 1/9 less than ordinary, and the

> abilities of 2 others 3/5 and 3/8 more, the work could have been finished 23/83 of an hour sooner. How many hours a day did the men work? (23:193)

This last one would today be considered a very ridiculous question. Who would measure time in 83rds of an hour? The problem itself, of course, is made up solely for the purpose of confusing the pupil and seeing whether he can get out of the maze of detail given.

A problem not quite so complicated but typical of many in early arithmetics dealt with rates of exchange.

> A merchant in Baltimore has credit for 4200 piastres at Leghorn, for which he can draw directly at 92 cents per piastre; but choosing to try the circular rout, he remits, first to Venice, at 24 piastres for 25 ducats, thence to Cadiz, at 320 mervadies per ducat; thence to Lisbon, at 630 reas per piastre of 272 mervadies; thence to Amsterdam, at 51d. Flemish per crusade of 400 reas; thence to Paris, at 56d. Flemish per crown; and thence to Baltimore at 54 cents per crown; what is the arbitrated price between Baltimore and Leghorn per piastre? and how much is the circular remittance better than the direct draft? (14:109)

Evidently there were tricks to the trade even in that day, for the answer indicated that the circular remittance was "best by 122 dol. 64c," the total sum amounting to $3,986.64, so the profit was evidently worth the effort.

Sometimes even the true values of numbers were disregarded in an effort to devise a question involving pure mathematical manipulation. Ostrander included one of these in the form of poetry:

> If the third of eight be four,
> What is the fifth of twenty score? (13:166)

It seems it would have been hard to develop in pupils any real sense of numerical values if problems of this type were used very often. Since the development of a feeling for numbers was one of the aims of arithmetic in the latter part of this study, such a problem would not be found in the more recent texts.

A problem which appeared over and over again in prose form as well as in verse, and was found in texts even as recently as 1898, concerned a chase of some kind. In Ostrander's text it was given in rhyme and dealt with the chase of a dove by a hawk. Sometimes a hare or a fox was being chased by a greyhound, or a thief by an officer. A typical example of this problem was found in Davies' text (1852):

> A hare is 25 of his own leaps before a greyhound, which is pursuing him. The greyhound makes 2 leaps while the hare makes 5; but 1 leap of the greyhound is equal to 3 of the hare's; how many leaps will the greyhound make before he overtakes the hare? (20a:198)

Few puzzle problems were found in texts in the late 1800's, but some of these are very interesting. Milne (1877) included one which on the surface seemed simple, but in reality was not so easy.

> A man engaged to work a year for $240 and a suit of clothes. At the end of 9 months an equitable settlement was made by giving him $168 and the suit of clothes. What was the value of the clothes? (32:128)

Although the puzzle element was challenging, problems were becoming less complex as time went on. The theory of mental discipline was still favored by many, but the tendency was to avoid trick questions and to include only those which would give the pupil practice in perseverance. An extreme example of this type was found in the Hornbrook series (1900):

> Separate the following into two lists, one of odd numbers, the other of even numbers. How many are there of each?
>
> 874; MDCCCLXXXVIII; the square of 7; the fifth multiple of 4; the product of 7 and 8; the quotient of 84 divided by 2; the difference between 81 and 18; the sum of 85 and 37; the largest number that can be expressed by two figures; the largest factor of 12 except itself; the number that is 5 greater than 212; the largest number that can be expressed by three figures; the smallest number that can be expressed by 3 figures; the number that means a dozen; the number that tells how many days in May; the integer between 17,345 and 17,347; one of the equal factors of 25; the factor that helps 7 to make 77; the square root of 100; the number that shows how many quarts in a peck; the denominator of the fraction 3/14; the greatest common divisor of 6 and 8; the remainder after dividing 25 by 11; the smallest multiple of 7 that will contain 5; the least common multiple of 4 and 7; the number that is just half way between 30 and 50; the smallest prime number greater than 25; the largest prime number less than 25; the numerator of the fraction 22/31; the number that tells how many square inches in a square foot; the number that is just as much less than 15 as it is greater than 11; the average of 19, 20, and 21; the first composite number; the number that shows how many pounds in a ton; the number that shows how many cubic inches in a cubic foot; the number that shows how many sides a pentagon has; the largest prime number that can be written with two figures; the smallest prime number that can be written with three figures; the quotient of 13.14 divided by .06; 50% of 862; the largest prime factor of 102; the number that shows how many millimeters in a meter. (45b:94)

On the whole, textbook writers at this time were making a distinct effort to avoid including brain teasers. In the preface to the Woodward series (1898-99), for example, the authors stated that they believed stress should be put on mastery of the fundamentals rather than on "riddles," for the common processes were basic to all arithmetic. Textbook writers, in general, are of the opinion, however, that puzzle problems have a strong motivating force. Therefore, while they have not approved of including them as a part of the regular lessons, a few recent authors have given a page or so to trick problems solely for the sake of creating interest among those who like to juggle figures. The *Modern Life Arithmetics* (1928) included questions like the following under the heading "Some Puzzle Problems":

> Mr. Adams sells 30 melons at 3 melons for $1 and receives $10. Mr. Brown sells 30 melons at 2 melons for $1 and re-

ceives $15. Together, they get $25. One day these two men were away on business, and Mr. Clark agreed to sell their melons for them, 30 melons apiece. On the way to the store Mr. Clark reasoned thus: "If I sell 3 melons for $1 for Mr. Adams and 2 melons for $1 for Mr. Brown, that is selling 5 melons for $2." So he sold the 60 melons at the rate of 5 melons for $2 and received only $24. What became of the other dollar? (54c:195)

How can you get exactly 4 gallons into an 8-gallon jar if you have only a 3-gallon, a 5-gallon, and an 8-gallon jar to use as measures? (54c:196)

Think of a number; multiply it by 8; add 20; divide by 4; subtract twice the first number. You should have 5. Have you? (54c:197)

Here again are catch questions, the kind found in contests and mental alertness tests, and exactly the type found in the earliest of the arithmetics but frowned upon by authors for at least 100 years. However, they have reappeared with a different purpose—such problems are fun today for pupils who have learned their arithmetic well enough to be able to solve them, while in the olden days they were a part of the required work of every student.

The most persistent form of the puzzle type of question has been that of the inverse problem. This is the kind in which the facts given in the problem could not possibly be known unless the answer was already known. Arithmetic as taught today has a purpose—children are made to feel the need for it and consequently to desire to solve the problems. Since there is no need for solving an inverse problem, the pupil often considers such a question silly and feels no urge to solve it. Authors have made a determined effort to weed out material of this kind, but inverse problems still creep in now and then because they offer an excellent means of testing the pupil's understanding. The type of problem will be recognized easily from the following:

A man gave away 3 dollars, which was 2 fifths of all the money he had; how much had he? (9:104)

If my horse and my chaise be worth $220 and the value of my horse be 2/5 that of my chaise, what is the value of each? (10:81)

A father's age and a son's age added together amount to 138 years. Twelve years ago the father was twice as old as the son. How old is each now? (41b:537)

A man drew out of the bank 1/9 of his money and $16 more; then ¼ of the remainder less $20. He then had $182 remaining in the bank. How much had he at first? (46b:380)

A forgetful merchant was the subject for a problem in Guthrie's text (1810)—a problem that would have been a horrible example of an inverse problem had the merchant remembered the details of his purchase. This is one way around the seeming uselessness of an inverse problem, for it was necessary here that someone help the merchant out by giving him the information he needed.

A merchant laid out £691 4 in cloths, but forgot the number of pieces purchased, and also how many yards there were in each piece and what they cost him per yard, but remembers

> that they cost him as many shillings per yard as there were yards in each piece, and that there were just as many pieces; required the number of pieces? (14:176)

According to Guthrie, the answer to this problem, 24, could be "found by reducing the money into shillings, and extracting the cube root."

Questions at foot of page.—As has already been mentioned, during the middle 1800's questions in a number of texts were placed at the bottom of the page. These at first dealt solely with material presented on the same page as the question—a carry-over from the catechism form of question.

Adams' New Arithmetic (1827), one of the first to include questions of this type, always listed the number of the article, or section, on which they were based, so that the pupil would have no difficulty in finding the answer. This section was always on the same page as the questions that were based on it. The group of questions on ¶31 was as follows:

> In what way is the first example performed? How might the operation be shortened? How often is one number contained in another? What, then, is division? Show its relation to multiplication. What is the object in the first, and what in the second, example? Define division. What is the dividend? to what does it answer in subtraction, and to what in multiplication? What the divisor and to what does it answer in subtraction and multiplication? What the quotient and to what does it answer? Explain the divisor and dividend in the first use of division. The dividend and quotient in the second. (17:49)

In this section a detailed solution for the first example was given, and an explanation was made as to how the operation could be shortened. Then followed a discussion of division as a "short way of performing many subtractions of the same number" (17:48), a definition of minuend, quotient, and so forth, and an explanation of the relation of division to multiplication. The pupil was thus given his review immediately upon studying his rules and definitions. Often the pupil was asked to repeat the rule or definition, sometimes even to repeat a table.

Greenleaf's text (1842) used this same type of question, carefully identified according to the section upon which it was based, but the questions much more often began with "How" and "Why." For example, on Sections 108 and 109 the questions were:

> 108. What is division of compound numbers?—109. Where do you begin to divide? Why? When there is a remainder after dividing any one denomination, what must be done with it? (26:125)

Here is evidence of a little greater effort to make the pupil think, for he is to tell why certain facts are true and certain processes must be used, and not merely repeat the rule. Although Felter (1864) did not include many questions at the bottoms of the pages, he was among those who thought twice about the advisability of putting the ques-

tions on the same pages as the answers, for he believed there was much more chance that the pupil would try to think of the answer if he had to go to some trouble to look it up. So in *Felter's Arithmetical Analysis* questions were based on material presented earlier in the text. However, he apparently felt the student could not be entirely relied upon to find the answers for himself, so he included the numbers of the sections in which they could be found. For example, following Section 183 were the questions:

> What is a unit? (1.) What is the difference between a unit and a number? (1.) (2.) What is the difference between an abstract and a concrete number? (3.) (4.) Of what does Arithmetic treat? (5.) What is a quantity? (6.) What is a problem? (7.) What is the difference between simple and complex problems? (8.) (9.) What is an analytical step? (10.) What is an analysis? (11.) What is a rule? (12.) What is a sign? (13.) What is an axiom? (14.) (24:220-1)

It will be noticed that practically all of these could be answered by quoting a definition. Here and there in his text, Felter returned to the earlier method of asking such questions and then giving the answers in full immediately following the question.

It must not be assumed that most written problems in early texts were in the form of questions calling for the repetition of rules, definitions, and principles. Problems involving number were given more space than any other type in all the arithmetics of this study. These other questions, however, were considered an important phase of arithmetic in the early days, when the subject was taught principally through rote learning. They are especially interesting because they are so foreign to texts today.

Problems involving processes now dropped from texts.—A number of processes, such as alligation and permutation, have been dropped from texts, some of them long before the beginning of the twentieth century. They were probably included originally because they afforded the student an opportunity to juggle figures. Problems based on these rules were often very impractical, sometimes even impossible, so far as actual experience was concerned. While many of the principles are still taught, they are presented in their relationship to more useful rules and processes rather than as individual principles.

Problems in alligation were all similar to those in Walkingame's text, quoted on page 116 of this study. They always involved the mixing of articles of different values. Sometimes the pupil was to find the cost of the mixture, sometimes the values of the articles included, and sometimes the quantity of each ingredient.

Problems in permutation usually dealt with the number of different ways a group of men could sit around a table, the number of times a certain number of bells could be tolled in different order, or the number of ways the letters in the alphabet could be arranged differently. Most of the problems in permutation were found in the very early texts. They were always similar to the following:

> How many changes may be rung upon 12 bells; and how long would they be ringing but once over, supposing 10 changes

might be rung in 1 minute, and the year to contain 365 days 6 hours? (2:96)

Nine gentlemen met at an inn, and were so pleased with their host, and with each other, that in a frolick they agreed to tarry so long as they, together with their host could sit every day in a different position at dinner; pray how long, had they kept their agreement, would their frolick have lasted? (5:339)

How many changes or variations will the alphabet admit of? (5:339)

The answer to this last question, as given in the text, is "620448401733239439360000"!

Single position, according to Pike (1797), "teaches to resolve those questions, whose results are proportional to their suppositions; such as those which require the multiplication or division of the number sought by any proposed number; or when it is to be increased or diminished by itself a certain proposed number of times" (5:331). It was sometimes called the "rule of false" because it used just any number, not the right one, to find the number sought in the problem.

A, B, C and D spent 35s. at a reckoning, and being a little dipped, they agreed that A should pay 2/3, B ½, C 1/3, and D ¼; what did each pay in the above proportion? (5:332)

A, B and C, playing at cards, staked 324 crowns; but disputing about tricks, each man took as many as he could: A got a certain number; B as many as A and 15 more; C got a 5th part of both their sums added together; how many did each get? (8:204)

According to Daboll (1799), arithmetical progression was described thus:

Any rank of numbers more than two, increasing by common excess, or decreasing by common difference, is said to be in Arithmetical Progression.

So { 2, 4, 6, 8, &c. is an ascending arithmetical series:
{ 8, 6, 4, 2, &c. is a descending arithmetical series:

The numbers which form the series, are called the terms of the progression; the first and last terms of which are called the extremes. (8:194)

Both arithmetical and geometrical progression were favorites in the old textbooks. They not only lent themselves to considerable mathematical manipulation, but the answers often proved so surprising as to create an interest on the part of the pupil. It is quite probable that the motivating quality of such problems helped to make them popular with textbook writers. All kinds of subjects were used, as will be noted from the following:

What sum would purchase a horse with 4 shoes, and 8 nails in each shoe, at 1 mill for the first nail, 2 for the second, 4 for the third, &c. doubling to the last? (14:166)

A lady, wishing to purchase 10 yards of silk for a new dress, thought $1.00 per yard too high a price; she, however, agreed to give 1 cent for the first yard, 4 for the second, 16 for the third, and so on, in a fourfold ratio; what was the cost of the dress? (26:297)

A sets out from one place, and B from another, 360 miles distant from A: they travel towards each other, so that A performs the first day 40 miles, the second 38, the third 36, and so on, decreasing his rate 2 miles daily. B begins the first day, and travels 20 miles, the second 22, increasing his rate 2 miles every day; in how many days will they meet? (11:207)

How many strokes does the hammer of a clock strike, in twelve hours? (8:195)

Suppose 100 apples were placed in a right line, 2 yards apart, and a basket 2 yards from the first; how far would a boy travel to gather them up singly, and return with each separately to the basket? (16:183)

This last problem was very popular with textbook writers. Sometimes potatoes or eggs were used, and the distances were different but the principle did not change and even the wording varied but slightly.

Sometimes problems in progression had a slightly more practical trend, as in:

There is a debt to be discharged at 16 several payments, arithmetical proportion; the first payment is 14 dollars and the last 100; query the whole debt and common difference? (14:175)

However, the answer to this problem "debt 922 dollars and common difference 15 dollars 73 cents 3+ mills," shows that here again was an example of the use of problems merely to exercise knowledge of a process, since it would of course be impractical to give payments in this form.

In some problems the terminology was not the same as that used today for the simple reason that the business practices on which the problems were based are no longer in use. Problems involving tare and trett and barter are examples of these:

What is the neat weight of 9 hhds. of Tobacco, each weighing gross 8 cwt. 3 qrs. 14 lb. tare 16 lb. per cwt.? (8:117)

Sold 12 butts of currants; each butt weighed 7 cwt. 1 qr. 10 lb. gross; tare 16 lb. per cwt. What was the amount at $9.20 per cwt.? (6:98)

Sold 3 casks of alum, each 675 lb. gross; tare 13 lb. per cent.—what are the neat weight and value at $4, 25 per cent. (16:111)

The answer to this last question was "1762 lb. neat—$74,87.4375: Or, 1762 lb. neat—$74,88.5 nearly [*sic*]." The comma was used by Daboll as a decimal point in dollars and cents, indicating the decimal parts of a dollar. However, he used a period to indicate any other decimal relationship, and therefore used it here to indicate the decimal parts of a cent. The term "nearly" was accepted by a number of authors in rounding out numbers.

Problems concerning barter dealt with a great variety of things. While only a few examples can be given, they indicate the general fields in which barter was carried on.

A man sold 98 gal. 3 qt. 3 pt. 2 gil. of whiskey at $1.37½ per gallon and took his pay in salt at ½ a cent per pound; how many barrels did he receive? (27:152)

C hath candles at 6s. *per* dozen, ready money, but in barter will have 6s. 6d. *per* dozen; D hath cotton at 9d. *per* lb. ready money—I demand what price the cotton must be at in barter; also how much cotton must be bartered for 100 dozen of candles? (2:173)

How many pounds of tea must be given for 28 pounds of rice, if 4 pounds of sugar are worth 1 pound of coffee, 15 pounds of sugar are worth 14 pounds of rice, and 30 pounds of coffee are worth 7 pounds of tea? (30:260)

Why the sugar and coffee had to be involved in this last problem is a question that would be hard for modern textbook writers to understand. They evidently were put in for the sake of adding to the difficulty and giving the pupil three problems in one.

A number of other topics in arithmetic that have dropped out of use and therefore out of the textbooks in the field could be listed here. However, the examples given provide a rather clear picture of the types of problems found in old texts and the subjects with which they dealt.

One other phase of written problems in old texts that perhaps should be mentioned before the most recent trends are discussed is that of using problems concerning what would today be called immoral practices. Prohibition days have of course been greatly responsible for the fact that references to spirits of all kinds are no longer found in texts. General disapproval of using as problem subjects anything that might suggest cheating, thievery, or even smoking has also been influential in the omission of such subjects. Some problems in early texts are rather shocking to one who has been used to finding subject matter based only on sober and fair dealings. A few of the problems that perhaps would not be found in modern texts were:

Divide 4½ gallons of brandy equally among 144 soldiers. (8:61)

A Merchant bought 8 Tuns of Wine, which being Sophisticated, he Selleth for 400 *l.* and loseth after the rate of 12 *l.* in receiving a 100 *l.* Now I demand to know how much it cost him *per* Tun? and how he selleth it *per* Gallon to lose after the said rate? (1:199)

How many chances are there in throwing 4 dice? (5:342)

A man smokes 3 cigars a day, at 10 ct. each; how much will they cost in a year of 365 days? (27:24)

How much water must be mixed with milk worth 10 cts. a quart, that it may be sold at 8 cts. a quart without loss to the seller? (31b:263)

This last question was found in a text published about 1875, so the attempt to exclude dishonest practices from business problems was evidently not thoroughly effective until later than that.

Ridiculous Problems

The only other important type of problem abandoned by more recent authors is what may be termed the ridiculous problem. It is somewhat difficult at times to distinguish between problems that were truly ridiculous at the time they were included in texts and those

that merely seem to be absurd because they do not fit in with conditions as they are today. Some of the examples already mentioned as representative of some special type of problem could easily fall into this group if it were not remembered that times have changed and that these problems, at the time they were used, had a sufficiently strong element of practicality to make them reasonably satisfactory.

Sometimes the ridiculous element in the problem was that the measure used was not divisible by the figure given in the problem, for example:

> A fox runs 3360 4/11 feet in 120 seconds; at that rate, how far does he run in one second? (24:130)

Since feet are not measured in elevenths, the distance a fox runs would never be given in this form. If the author had wanted to test the pupils' ability to handle fractions of this denomination, he should have used a problem in which such a division of subject matter would have been practical. The same kind of thing is true in:

> A man, dying, left an estate of $33463 4/15 to be equally divided between his four children after his wife had taken a third; what did each child have? (24:131)
>
> Three merchants, A, B, and C, put money together, and gained 1000 dollars, of which A took a sum unknown, B took twice as much as A, and C thrice as much as B;—Query, the share of each? (14:160)

The first of these two problems was given in fifteenths of a dollar and the answer to the second is in ninths of a dollar. Even in the days when United States currency included mills, dollars were not divisible by fifteenths or ninths. A slightly different type of impracticality in problems involving money is found in the following:

> A building valued at 10520D.78 cts. is insured at 12½ cents per cent.; What is the amount? (7:113)

As everyone knows, no building worth more than $10,000 would have its value given in such detailed form as this. Even an odd number of dollars beyond the $500 would be unusual, and certainly the 78 cents is ridiculous.

Sometimes the pupil's knowledge of tables of measures would be tested through problems in which measures were involved that would not be used in actual life, for example:

> If 5 children at a picnic eat 21 *hectogrammes* of cake, how much will it take to feed 93 children? (28:198)

In the measurement of time the following was found:

> When I arrived in Chicago I found my chronometer 1 h. 6 min. 52 sec. slow; how many degrees of longitude, and from what direction had I come? (27:154)

No traveler would find his watch a certain number of hours, minutes, and seconds slow, since changes in standard time are based on hours only.

Sometimes the figures used in problems were so large or so involved as to be out of all reason so far as practicality is concerned. Among these were:

> What is the sum of 30 terms of the powers of 3? (11:208)
> Reduce $\frac{1}{2048000}$ to a decimal. (20b:219)
> If John's uncle had given him $\frac{2222222}{4444444}$ of a dollar, how many cents would he have received? (45b:115)

Surprisingly, this last problem was found in a text published less than 50 years ago. All these problems were foolish from the point of view of their useless manipulation of figures. Their purpose, of course, was to give drill in handling numbers rather than to develop an understanding of the use of figures in daily life.

The ridiculous element in some of the problems was in the fact that, when the answer was obtained, there was no use for it. In some, no one would ever perform the act mentioned, for example:

> How long would you be in counting $3000000 in half-dollars, if you counted 5 half-dollars per second, and worked from 7 o'c. A.M. till 12 M., and from 2 o'c. P.M. until 6 P.M.? (25:318)
>
> If the earth be 360 degrees round, each degree 69½ miles, how long would a man be travelling the circumference, at 20 miles a day, admitting there were no obstacles, and reckoning 365¼ days in the year? (14:174)

This last problem was found in Guthrie's text under the heading "Practical Questions." According to the answer, the man would walk for 3 years and 155½ days, at the rate of 20 miles a day. It seems strange that an author would be so careful about details as to give the number of days in a year as 365¼ instead of rounding the figures out to 365 and should remember to mention the fact that all obstacles had to be considered as eliminated, and yet should ask such a ridiculous question.

A popular question, found in many texts and in many forms, was the kind that asked for the number of minutes in a certain number of years. Usually these problems concerned the age of some person, or the number of minutes in a certain era, but, since time in such expanse is always given in terms of years, there is no object in determining the number of minutes involved. Nevertheless, such questions as the following appeared time and again:

> How many minutes are there in 7 signs of the Zodiac? (7:74)
>
> How many minutes have elapsed since the creation of the world to the year 1806, inclusive? (7:194)

The year 4004 B.C. was taken as the date of the creation of the world in this problem, the answer being 3,055,827,600 minutes!

Another problem in which the answer would be of little use is:

> What weight of water may be contained in a canal whose depth is 8 feet, width 25 feet, and length 12 miles? (23:31)

Knowledge of the weight at any one point, or the volume of the whole, might be of some use, but why should anyone need to know the weight of all the water in the canal?

Sometimes the ridiculous part of a question was in the fact that the information given in it was entirely out of line with ordinary practice. Although many such problems appeared in the texts, even in comparatively recent years, somewhat of a risk is involved in selecting them because conditions 150 years ago differed so greatly from those today, and what might have been considered right in those days sometimes appears very peculiar today. Therefore only a few of the most extreme problems of this type will be mentioned. One of the favorites concerned a circular garden:

> A circular garden contains 2 acres; required the length of a stone wall which will enclose it. (13:170)

No one is going to build a circular garden of this size, and it is doubtful whether anyone ever did.

Many of the old arithmetics contained questions about dressmaking or tailoring which seem ridiculous today, but in those days dresses were made very full and took much more material than they take today. Although the figures in the following problems were probably fairly accurate, they are amusing:

> What sum did that gentleman receive in dowry with his wife, whose fortune was her wedding suit: her petticoat having two rows of furbelows, each furbelow 87 quills, and each quill 21 guineas? (2:26)

> From a fashionable piece of cloth, containing 48 yards, a tailor was ordered to make 4 suits, each seven and a half yards, how much remains of the piece? (14:30)

In some cases, however, the figures used could not have been right even in olden times. Take the following, for example:

> If 10 men can cut 46 cords of wood in 18 days, working 10 hours a day, how many cords can 40 men cut in 24 days, working 9 hours a day? (46b:407)

According to this problem, it would take each man nearly 40 hours to cut a cord of wood! Another problem that sounds almost as ridiculous, though its absurdity is of an entirely different type, is:

> A coat that cost me $17.40 was sold for $1.50; how much did I lose? (27:21)

The assumption is that the owner of the coat did not get any wear out of the coat. If it were still a new coat there seems to be no logic in his having sold it for only $1.50.

Prices were very different from those today and, not only that, they varied greatly from book to book and sometimes from problem to problem within the same book. Now and then a problem was found that sounded too absurd to have been within the realm of possibility, like:

> 48 head of poultry bought at 5 cents per head; how many cents to pay? (11:101)

But oftentimes prices of the day accounted for many of the seeming absurdities. A few of the prices found in texts before 1850 were as follows:

Beef	4¢ to 7½¢ per lb.
Butter	9¢ to 27¢ per lb.
Cheese	4½¢ to 18½¢ per lb.
Coffee	11½¢ to 44½¢ per lb.
Corn	25¢ to 87½¢ per bu.
Flour	$5 to $8 per bbl.
Ham	55¢ per lb.
Hats	$1.25 to $4.25 each
Hose	87½¢ to $1.25 per pr.
Land	$13.67 to $14.67 per A.
Lard	15¢ per lb.
Linen	55¢ per yd.
Muslin	11¢ to 37½¢ per yd.
Paper	$2.35 per ream
Pencils	5¢ each
Pork	8¢ per lb.
Potatoes	56¼¢ per bu.
Satin	$3.75 per yd.
Shoes	$1.51 per pr.
Sugar	6½¢ to 12½¢ per lb.
Textbooks	6¢ to 37½¢ each
Wheat	50¢ to $1.25 per bu.

This is just a sampling of the commodities referred to in early arithmetic problems. However, it is enough to show that, while in some cases prices were very different from those charged today, in the case of 5-cent pencils and a few other things, for instance, they have remained the same through the years.

Throughout the problems listed here as representative of ridiculous problems found in textbooks there has been the same deep-seated purpose—that of providing drill material for pupils. With such an objective, the emphasis has been placed either on the thought or on the facts of the problem, without correlating the two so that the statement would make sense. Authors tried to clothe their figures with varying subject matter in order to hold the interest of the pupils, and in so doing sometimes stepped over the line into a field entirely inappropriate for the figures or the processes used.

Problems Typical of Recent Arithmetics

Seemingly, recent authors have been much more aware of the value of written problems in teaching the child to think through a situation and to apply his knowledge of arithmetic to the affairs of everyday life than were the earlier textbook writers. They also showed more interest in making problems attractive to children, in arousing in the pupil the feeling of need for such information, and in keeping the child interested from the beginning of the text to the end.

In accomplishing this aim, authors have made changes in written problems, so that the traditionally accepted subject matter and form of the problem are no longer the only ones used. Changes have been greatly influenced by the testing movement, which has produced different types of problems and has attacked them from different points

of view. Then, too, problems have been correlated with other subjects and have been put in a storybook form, so that the child has an entry to his arithmetic through other fields of knowledge and through his interest in the activities of a certain family group or community group. Such problems have been put on the level of the child's comprehension and interests and have often become very fascinating to him because of factors not a vital part of the arithmetic itself.

Changes in subject matter.—New subject matter has, of course, been added as new inventions or customs have developed. Problems like the following, for example, would never have been found in old texts:

> The radiator in Mr. Gregory's car holds 4 gal. How much alcohol must be bought to fill it with a solution that will not freeze at 20° below zero? (58f:155)
>
> Mr. Coe was married and had two children under 18. The exemptions are $1000 for a single man, $2500 for a married man, and $400 for each dependent child. Find Mr. Coe's total exemption. (58f:137)

However, there is nothing truly new about the problems themselves. The processes involved are the same as those in older texts, but the difference lies in the fact that these problems deal with present-day situations, with things that go on every day in the lives of the children and that they therefore want to know more about.

Changes in form.—Not until after 1890 was much evidence found of attempts to vary the form in which written problems were stated. The usual method had been to state certain facts and figures and to ask what results would be obtained when these facts and figures were used for a certain purpose. Each problem was complete within itself and did not necessarily have any relationship, other than process involved, to problems either above or below it on the page.

The first attempt to use completion problems was found in the elementary text of the Southworth series (1893), where such problems as the following were included now and then:

> Use one of these numbers to fill the first blank:—10, 20, 15, 35, 40, 25, 30, 45, 50.
>
> When 5 pints of oil cost —— cents, one pint will cost 1/5 of —— cents, or —— cents; 3 pints will cost 3 x —— cents, or —— cents. (40a:24)

Not many such problems were found in the text, however, and other authors did not take up with the idea very well. Nevertheless, the fact that it was a clever attempt to break away from the traditional form deserves recognition.

A great effort was made about this time to relate the child's school life with outside experiences, and teachers were asked to create situations in which children could see the relationship of their arithmetic to life. This was evident in problems relating to other subjects in the curriculum and in those in which figures available from the children themselves could be used in making up problems, such as:

——— may give the date of his next birthday, and the class may find the number of days intervening. (45b:167)

Such problems developed into the idea of having pupils make up their own problems, a plan which helped the pupil see his arithmetic from a different point of view and therefore to get a clearer conception of the meaning and value of arithmetic problems. In several of the texts, the Hornbrook series, for example, pupils were given the opportunity to make story problems for their classmates to solve and, even as late as the *Daily-Life Arithmetics* (1938) the same idea was used, but usually with background material already supplied.

Make problems to find how many were left.
1. 6 pencils on the desk; 4 rolled off.
2. 9 pieces of paper; 3 were thrown away.
3. 8 shoes; Don's dog Spot carried away 5.
4. 10 tablets; 3 tablets were used up. (58a:8)

This problem was designed for beginners in their first few lessons in arithmetic. The authors offered a helping hand by suggesting that the pupils play the problem if they found it hard. This was typical of most modern books. Wherever new material was taken up or problems were presented from a slightly different angle, suggestions were made to the pupils as to how they could help themselves to find the answer. Questions, too, did not always use numbers or call for a numerical answer. Sometimes pupils were asked how to do a problem, or why a certain answer was incorrect. Examples of these were found especially in the most recent of the arithmetics.

If you know the number of pecks and the price per bushel, how can you find the cost? (50b:164)

If you know the amount of a bill and the rate of discount, how do you find the net amount of the bill? (52b:195)

Mary picked strawberries for her father. There were 12 equal rows in the patch. By noon she had picked 8 rows and had 45 quarts. How many quarts should she get from the whole patch at that rate? How many different ways can you find to work this problem? (57e:67)

Ted thought he would get 6 pieces of candy when Fred said he would give him 2/5 of his 15 pieces. Was Ted right? (58b:87)

Why is 7/12 a foolish answer for Problem 7 below?
The owner of the dairy told Helen that one cup of milk contains as much lime as 7 eggs or 42 slices of bread contain. A dozen eggs contain approximately as much lime as ——— cups of milk. (59d:222)

In these problems the pupil was tested on his understanding rather than on his ability to obtain a correct solution to a problem. He had to know what he was trying to find out and how he should go about it before he could answer such a question as the last one listed above, for example. If his knowledge was based on rote learning, he could not answer this question correctly, for it called for clear thinking and understanding. It is this that is being sought in the schools of today, and the movement is clearly reflected in the written problems of arithmetic texts.

The "story" form of problem is comparatively new although units of problems, all dealing with the same group of persons or the same general activity, have been popular for some time. When the unit plan is used a central theme runs through a group of problems covering from a page or so to an entire book. At first units were merely a group of problems held together in thought by some factor other than the process or principle being studied. Later, problems in units began to develop out of stories that were told, so that the child had an opportunity to see the need for his knowledge of arithmetic in following the story intelligently. In some cases, where problems were not in units, a story was found within each problem, so that it was more interesting to a child than a similar problem in the old style would be. For instance, instead of asking a simple question about how many puzzles a girl would give away if she gave six to each of two friends, a little story was made of the problem in the *Study Arithmetics*:

> Betty cut 18 puzzles out of a newspaper. Ann and Nancy liked the puzzles. So Betty gave each of the girls 6 puzzles. How many puzzles did Betty give to Ann and Nancy together? (59a:185)

Here the children in the problem became real because they had names; the puzzles could be more clearly visualized because they were described as the kind that were found in newspapers; and the action took place for a reason, because Ann and Nancy liked the puzzles, not just because something had to happen in order to make a problem.

The *Daily-Life Arithmetics* were especially successful in creating interesting situations for problems with a true storybook air. In the story of "Peter's Farm," for example, a sketch of the farm was at the top of the page. Then came an introductory paragraph:

> This is a map of the farm to which Peter and his family moved. Peter had never lived on a farm, and he wanted to write to his friends in the city to tell them about the big fields. But first he had to find out the areas of the fields. (58d:146)

This was followed by four problems, the first of which was:

> 1. Peter liked the pasture best of all. There were a few trees, grass, and a little stream there. The pasture was 30 rd. x 60 rd. He wanted to write, "We have a (?)-acre pasture." How many acres should he say? (58d:146)

The tone of the problems was personal, about something in which the child was interested because he saw the need for knowing more about it, yet the information taught was just as practical as that in dry, uninteresting problems of early texts. It is the belief of authors in the value of appealing to the child's interest and developing his enthusiasm that is causing texts to have more appeal in the subject matter of problems as well as in physical make-up.

Subject Matter of Written Problems

The subject matter of written problems seems to be worthy of further analysis, for it is believed a study of the things or persons about which the problems center will show to a great degree the

practicality of the problems and the extent to which they dealt with subjects within the experiences and understanding of the pupils.

The term "subject matter," as here used, refers to the situation or the people or the place involved in the problem rather than the principle or process. Through a study of subject matter, much may be learned about the development of the child's textbook out of the business man's "assistant."

In determining the subject matter of problems, only the concrete nouns were studied, as it seemed these would give a fairly clear picture of the subjects with which problems dealt. Some problems, of course, included a number of such nouns, sometimes in different classifications (as in the case of barter), and were therefore represented more than once. However, the subjects rather than the problems were the things being studied, and therefore it was considered logical to handle them as individual subjects.

Classification of nouns used.—In compiling the list of concrete nouns, groupings were made according to certain major classifications; that is, all words relating to foods and farm products were thrown into one group, those concerning cloth and clothing into another, and so forth. The number of different words used in each of these fields and the number of times they were used are shown in Table III.[9]

Of the 2,581 different nouns used in the texts, nearly 15 per cent were in the field of occupations. This seems to indicate that the texts were practical from the standpoint of offering problems in various vocations. However, only slightly more than 8 per cent of the total number of nouns used were in this field, a situation explained only by the fact that, while many different vocations were mentioned, few problems dealt with each vocation.

Just the opposite was true of foods and farm products, the classification in which the next largest percentage of different nouns was found. While more than 12 per cent of the different words were in this field, over 24 per cent of the total number of nouns used referred to foods or farm products. There evidently was much more repetition of terms here than in the field of occupations.

The table should be studied carefully from this point of view. These differences are especially evident in the case of chemicals, drugs, dyes, and minerals, a field in which there are many, many terms that could be used. More than 3 per cent of the different words used but less than a half of one per cent of the references to such subjects were in this field. The same was true of military items. On the other hand there are but few nouns relating to money and banking and to people, but these few were referred to much more often than were words in some other fields.

The trend toward an increase or decrease in the use of nouns in certain fields through the five periods of the study is also interesting. As might be expected, alcoholic beverages lost favor steadily as books were designed more and more for children, but this was the

[9] The data given in Tables III and IV were taken from material collected in the earlier analyses of arithmetics by 30-year periods, published by the Bureau. (See footnote, page 7.)

TABLE III. NUMBER OF DIFFERENT CONCRETE NOUNS USED IN EACH SUBJECT FIELD AND NUMBER OF TIMES SUCH WORDS WERE USED

Subject field	Number of different words used							Times words in each field were used	
	30-year periods					Total	Per cent	Number	Per cent of total
	1	2	3	4	5				
Alcoholic beverages	13	12	11	8	4	15	.58	828	.75
Animals, birds, fish, insects	6	21	30	57	98	127	4.92	1,175	1.06
Body references	0	3	3	9	4	13	.50	109	.10
Buildings, factories, stores	15	32	61	67	80	140	5.42	6,622	5.99
Chemicals, drugs, dyes, minerals	14	20	37	44	7	78	3.02	408	.37
Cloth and clothing	65	59	80	100	120	186	7.21	7,296	6.50
Communication	0	2	8	12	9	15	.58	384	.35
Construction terms and materials	42	79	139	181	175	287	11.19	10,333	9.34
Foods and farm products	98	123	160	211	261	321	12.43	26,822	24.24
Games and recreation	10	23	41	88	59	110	4.26	3,795	3.43
Geographic and astronomical references	8	15	15	23	11	30	1.16	1,806	1.63
Household and personal equipment	47	76	153	181	203	318	12.31	9,903	8.95
Metal articles and machinery	3	7	20	24	32	43	1.67	677	.61
Metals and alloys	17	13	21	23	11	30	1.16	1,213	1.10
Military items	22	24	34	35	31	63	2.44	631	.57
Mines, quarries, wells	1	2	4	5	1	6	.23	82	.07
Miscellaneous items	21	27	45	60	30	89	3.45	4,761	4.32
Money and banking	4	6	6	11	10	13	.50	4,227	3.82
Occupations	90	107	205	185	181	385	14.90	8,989	8.14
People	29	20	44	47	33	70	2.71	7,671	6.94
Plants, flowers, trees	1	4	11	20	10	28	1.08	838	.76
Political divisions	3	3	6	5	6	8	.31	799	.72
School terms and materials	11	22	41	48	68	85	3.29	6,264	5.67
Transportation	20	33	62	63	78	121	4.69	5,039	4.56
Total	567	733	1,129	1,507	1,522	2,581	100.00	110,672	100.00

only topic that decreased. A few remained fairly stationary, but most subjects, especially animals, birds, fish, insects; household and personal equipment; school terms and materials; and transportation were much more frequently found as subjects for problems as time went on. The figures must be studied in relation to the total, however, for there were only about one third as many different nouns used in the first period as in the last.

On the whole, Table III indicates that, according to subjects, problems seem to have been practical and within the range of everyday experience throughout the years covered by this study. While the number of different terms used was necessarily limited somewhat by the field in which they occurred, the number of times such terms were used was not so limited and can be taken as an indication of the popularity of certain fields as background for written problems. Foods and farm products were by far the most often used, construction terms and materials came next, followed in order by household and personal equipment, occupations, people, and cloth and clothing. No other classification contained more than 6 per cent of the references.

Comparison of subject matter of early and late problems.—A comparison of the wording of written problems in early and late arithmetics was especially interesting. In the discussion of the problems as a whole, unusual expressions, awkward statements, and even actual errors in grammar were evident in the older books, while in recent arithmetics, where the storybook form of problem was more often found, subjects became less technical and the vocabulary of the problems also reflected the characteristics of the times and the changing purposes of teaching arithmetic.

The types of change that have taken place in subject matter are particularly evident in Table IV, which gives interesting examples of words which have gained in use or which have lost in favor. Only a few of the words showing distinct tendencies to increase or decrease in popularity could be included, but these have been selected as representative. They have been listed according to subject field in order that comparisons may be seen more readily.

Beer, brandy, rum, and wine were especially popular as problem subjects in texts before 1850, but the campaign for prohibition and the increasing desire to write problems about subjects with which children were personally acquainted apparently had their effect, for no alcoholic beverage has increased in use in written problems. Words suffering loss of popularity for the same reason were the noun "tavern" in the group of buildings, factories, stores, and the words "vintner," "wine merchant," and "refiner" in the list of occupations.

Besides the group of subject nouns considered more or less taboo today, there are three other large classes of words dropping from use. The first of these is made up of words carried over from olden times, when castles, courtyards, moats, and knights were typical of the times. The second is a group of nouns that have become more or less obsolete. Examples of these are found especially in the field of cloth and clothing, where drugget, humhum, shalloon, and stuff were types of yard goods popular in early times, and in the field of

TABLE IV. TWO HUNDRED EXAMPLES OF WORDS SHOWING A DEFINITE TENDENCY TO DROP OUT OR TO INCREASE IN USE

Words decreasing in use						Words increasing in use					
	30-year periods						30-year periods				
Word	1	2	3	4	5	Word	1	2	3	4	5
ALCOHOLIC BEVERAGES						ALCOHOLIC BEVERAGES					
Beer	15	31	9	1	0						
Brandy	41	40	13	3	0						
Rum	15	16	1	0	0						
Wine	128	146	111	38	0						
ANIMALS, BIRDS, FISH, INSECTS						ANIMALS, BIRDS, FISH, INSECTS					
Greyhound	2	8	2	1	0	Cat	0	0	1	5	22
Snail	2	1	0	0	0	Elephant	0	0	2	1	18
						Fish	4	14	23	40	124
						Pony	0	0	1	22	35
						Puppy	0	0	0	0	18
BUILDINGS, FACTORIES, STORES						BUILDINGS, FACTORIES, STORES					
Castle	2	4	1	0	0	Cafeteria	0	0	0	0	14
Lighthouse	4	1	0	3	0	Garage	0	0	0	0	53
Steel yard	1	1	0	0	0	Home	0	0	3	15	256
Tavern	0	3	2	0	0	Hotel	0	0	8	10	39
						Station	0	0	3	5	45
CLOTH AND CLOTHING						CLOTH AND CLOTHING					
Baize	5	2	0	0	0	Cap	0	0	6	13	50
Dowlas	3	0	0	0	0	Dress	2	3	10	68	159
Drugget	7	2	2	0	0	Handkerchief	1	5	7	25	60
Holland cloth	17	0	0	0	0	Hose, hosiery	1	2	3	13	22
Humhum	0	4	0	0	0	Pocketbook	0	0	0	4	17
Pantaloons	0	5	6	1	0	Ribbon	9	14	23	246	270
Shalloon	11	4	0	0	0	Shoe	8	25	40	128	141
Stockings	14	11	5	1	2	Suit	4	8	15	57	159
Stuff	8	0	0	0	0	Sweater	0	0	0	0	57
Worsted	2	2	0	0	0						
COMMUNICATION						COMMUNICATION					
						Letter	0	0	2	13	92
						Stamp	0	0	1	8	69
						Telephone	0	0	0	12	51
CONSTRUCTION TERMS						CONSTRUCTION TERMS					
Courtyard	5	2	1	1	0	Concrete	0	0	0	0	38
Moat	2	3	0	0	0	Floor	5	26	54	69	203
Partition	2	1	1	1	0	Furnace	0	0	0	3	16
Water wheel	3	0	0	0	0	Hinge	0	0	1	3	12
						Living room	0	0	0	0	28
						Tank	0	0	5	121	185
						Tool	0	0	2	1	15
						Wallpaper	0	4	6	5	18
						Window	3	7	43	39	139
						Wire	1	2	2	28	83
FOODS AND FARM PRODUCTS						FOODS AND FARM PRODUCTS					
Broom cane	3	0	2	0	0	Apple	11	189	245	478	521
Fig	9	6	5	1	0	Banana	0	0	0	52	84
Heifer	0	3	0	0	0	Cereal	0	0	0	1	11
Provisions	7	42	20	9	0	Cream	0	0	1	7	81
Tobacco	60	47	12	8	10	Cookies	0	0	0	2	123
						Egg	3	22	85	360	445
						Fudge	0	0	0	0	26

TABLE IV (Continued)

Words decreasing in use						Words increasing in use					
	30-year periods						30-year periods				
Word	1	2	3	4	5	Word	1	2	3	4	5
						Grapefruit	0	0	0	1	51
						Ice cream	0	0	0	1	88
						Ice cream cone	0	0	0	0	38
						Jelly	0	0	0	13	17
						Milk	0	8	51	263	590
						Orange	4	7	67	270	325
						Pie	0	1	2	11	100
						Popcorn	0	0	0	1	41
						Potato	3	26	101	249	351
						Sandwich	0	0	0	0	74
						Tomato	0	0	4	21	93
GAMES AND RECREATION						GAMES AND RECREATION					
Lottery ticket	1	4	1	0	0	Baseball	0	0	0	18	143
Maypole	2	1	0	0	0	Basketball	0	0	0	0	47
						Bicycle	0	0	0	136	166
						Camp	0	0	0	0	69
						Doll	0	0	4	47	170
						Garden	5	9	32	62	129
						Kite	0	0	8	31	46
						Movies	0	0	0	0	36
						Playground	0	0	2	2	71
						Radio	0	0	0	0	71
						Skates	0	0	10	11	76
						Swimming pool	0	0	0	1	32
HOUSEHOLD AND PERSONAL EQUIPMENT						HOUSEHOLD AND PERSONAL EQUIPMENT					
Beeswax	3	0	0	0	0	Bookcase	0	0	3	14	36
Chaise	3	10	8	0	0	Bottle	0	13	12	57	139
Porringer	2	0	0	0	0	Box	1	10	29	340	502
Salver	2	0	4	0	0	Calendar	0	0	0	1	28
Snuffbox	1	2	0	0	0	Can	0	0	1	57	225
Tallow	11	6	6	3	0	Gas	0	0	0	29	63
Tankard	11	7	4	0	0	Linoleum	0	0	0	6	28
						Paper	11	14	74	356	468
						Refrigerator	0	0	0	5	24
						Rug	0	0	1	38	64
						Shelf	0	1	2	4	73
						Table	0	4	7	50	166
						Thermometer	0	4	5	9	51
						Towel	0	0	1	5	29
METAL ARTICLES AND MACHINERY						METAL ARTICLES AND MACHINERY					
Cock	9	12	0	0	0	Engine	0	2	8	17	43
						Machine	1	0	1	11	35
						Sewing machine	0	0	5	9	14
						Tractor	0	0	0	0	15
MISCELLANEOUS ITEMS						MISCELLANEOUS ITEMS					
Auction	2	0	2	0	0	Cardboard	0	0	0	15	43
Crown	0	3	0	0	0	Flag	1	0	4	14	42
Pendulum	6	6	0	0	0	Home	0	0	3	15	256
Rations	25	51	0	0	0	Ice	0	0	7	30	54
						Ranch	0	0	0	1	31
						Sidewalk	0	1	7	22	81
						Ticket	0	2	3	2	225
						Water	20	32	87	337	431

TABLE IV (Continued)

Words decreasing in use						Words increasing in use					
	30-year periods						30-year periods				
Word	1	2	3	4	5	Word	1	2	3	4	5
MONEY AND BANKING						MONEY AND BANKING					
						Bank	6	11	75	326	574
						Check	0	1	0	8	139
						Money order	0	0	0	16	23
OCCUPATIONS						OCCUPATIONS					
Brewer	2	0	4	0	0	Aviator, pilot	0	0	0	0	19
Clergyman	0	1	6	0	0	Clerk	5	5	18	105	180
Correspondent	11	0	1	4	0	Contractor	0	2	8	19	51
Courier	1	1	2	0	0	Dealer	2	0	23	305	451
Draper	15	6	1	0	0	Driver	0	0	1	1	19
Factor	14	0	4	0	0	Fireman	0	0	0	0	16
Footman	5	1	0	0	0	Salesman	0	0	4	6	64
Goldsmith	17	14	1	0	1	Teacher	0	2	14	124	140
Guardian	6	0	0	0	0						
Privateer	9	2	0	0	0						
Refiner	6	0	1	0	0						
Schoolmaster	2	2	0	0	0						
Tobacconist	7	1	2	1	0						
Usurer	5	2	0	0	0						
Vintner	14	8	6	0	0						
Wine merchant	10	1	7	0	0						
PEOPLE						PEOPLE					
Friend	9	0	2	0	0	Boy	6	2	52	192	945
Gentleman	57	46	51	19	0	Brother	0	0	1	19	160
Knight	2	0	0	0	0	Family	14	29	28	19	276
Pauper	0	2	0	0	0	Girl	0	3	19	64	439
Ward	2	0	0	0	0	Grandmother	0	0	1	0	42
						Uncle	2	0	0	2	66
POLITICAL DIVISIONS						POLITICAL DIVISIONS					
						City	1	4	12	37	338
						County	0	0	0	0	24
						State	0	0	3	18	77
						Town	5	3	20	34	143
SCHOOL TERMS AND MATERIALS						SCHOOL TERMS AND MATERIALS					
Geometry book	0	2	1	0	0	Blackboard	0	0	2	93	176
Quill	3	15	4	0	0	Class	0	1	11	34	338
Scholar	3	15	4	7	0	Crayon	0	0	2	15	35
						Eraser	0	0	1	5	35
						Map	0	0	2	15	79
						Pen	0	1	17	41	52
						Pencil	0	23	28	204	253
						Pupil	3	4	38	241	574
						Ruler	0	0	2	7	83
						School	2	9	49	208	757
						Schoolroom	0	0	7	91	160
TRANSPORTATION						TRANSPORTATION					
Axletree	0	3	0	0	0	Airplane, plane	0	0	1	3	182
Plank road	0	1	3	0	0	Automobile	0	0	0	14	322
Post road	0	2	0	0	0	Bus	0	0	0	0	50
Stage	3	2	2	0	0	Elevator	0	0	0	8	20
Team	1	3	6	0	0	Street car	0	0	3	9	20
						Train	0	1	43	167	305
						Truck	0	0	0	4	91
						Tunnel	0	0	2	0	257

household and personal equipment which included such things as porringers, salvers, and snuffboxes. In the last of the main classes of words no longer found in textbooks are those that deal with things the child seldom meets in life, such as greyhound, pendulum, and axletree.

Only two of the words listed in Table IV among those being dropped from texts, "stockings" and "tobacco," were found in books of the last 30-year period. A few words, like "Holland cloth" and "water wheel," did not appear after 1820, and most of those in the list disappeared before the end of the century. Tobacco was one of the subjects many authors thought best to omit from textbooks for children. While it is seldom used today, it was formerly among the most popular problem subjects and is still found in modern texts.

Change in style or in usage has accounted for several of the differences in terminology and consequent seeming omission or addition of a subject. For example, the term "pupil" has replaced "scholar," "teacher" has replaced "schoolmaster," "pen" has replaced "quill," and "automobile" has replaced "carriage." Popular terminology has been responsible for other changes, too, not shown in the table; words such as "engine" for "locomotive," "shoes" for "boots," and "basement" for "cellar" are among the more interesting of these.

The words increasing in use during the 150 years show much more startling figures than do those decreasing, but it must be remembered that there were ten times as many nouns used in textbooks of the fourth 30-year period and more than 12 times as many in textbooks of the fifth period as in those of the first. When this is taken into consideration, these figures do not seem so extreme.

As in the case of the words dropping from use, the nouns in Table IV may also be grouped into certain classifications, the three principal ones being words that deal with things or activities that have come into existence during the past century and a half; those that deal with specific things, such as special foods instead of just the term "food"; and those that deal with things of interest to children.

Many words have been added to the English vocabulary in the last 150 years—cafeteria, for example, and garage, telephone, ice cream, radio, airplane, automobile, and elevator. These are representative of an important part of daily life, and children have contact with them often. For this reason they should be used as problem subjects. Textbook writers are always slow about accepting new terms, and it is often a number of years after a word is in common usage before it is found in a text. Blackboards, for instance, were used early in the 1800's, yet they were not mentioned in the written problems of arithmetics until after 1850. Even then the word appeared only twice before 1880, but after that date it was commonly used.

The attempt to make problems more real to children has led authors to deal with specific rather than general things. A problem is more or less abstract in content when it deals with animals, or food, or merchants but when it deals with dogs and cats, ice cream and cake, or grocers and druggists the child can picture the situation and can see the relationship of his problem to the things he meets in

everyday life. The use of the word "home" is a good example of the attempt to give problems a personal meaning, for this word was not found in texts before 1850 ("house" had usually been used) but was found 256 times in the last 30-year period.

The tendency to use specific subjects is especially noticeable in the field of cloth and clothing and of foods and farm products. Most of the words dropping from use in the cloth and clothing group refer to yard goods, a subject which children do not know much about, while those coming into use refer to dresses, sweaters, shoes, and suits, things with which children come in contact in their very early years. In the foods group, each fruit and vegetable is mentioned separately, and stress is placed on foods that are wholesome, like cereal, eggs, and sandwiches, and those that are particularly enjoyed by children, like cookies, fudge, and popcorn. Milk and cream rose in popularity as alcoholic beverages lost favor. Milk made the greatest gain of any subject, as it was not found at all in the first 30-year period but appeared 590 times in the last.

In making problems interesting to children, many of their activities, like baseball and skating, were used. Bicycle, puppy, doll, pony, and many other things prominent in children's activities increased tremendously in use as problem subjects. People taking part in the activities became real people. No longer were they "a gentleman" or "a friend," they were other members of the family, or boys and girls with real names.

In three divisions, communication, money and banking, and political divisions, none of the terms used dropped from use in written problems, but this was principally because all three fields have become increasingly important in the life of the country during the period studied. With the coming of the industrial age and the many inventions which accompanied it, means of communication increased so rapidly and were used so extensively that it was natural they should become the subjects of many textbook problems in arithmetic. In early books money and banking had been handled purely as a phase of big business—bankers, brokers, and merchants were those about whom most problems in this field were usually written. Later, however, banking as it related to the welfare of the child, in teaching him to save and to handle money in a businesslike manner, became recognized as an important phase of his education, and was taught as a part of his arithmetic study. Probably the greatest factor in the increase of words used in the field of political divisions was the growing interest in correlating arithmetic with civics, geography, and history. Units in these fields often contained problems dealing with political divisions.

Besides the words which remained fairly constant in their popularity and those which showed a tendency to drop out or come into use, there were a few which came into vogue after 1790 but which showed signs of dropping out again before the end of the study. Much the largest proportion of these words were the general, all-inclusive terms, such as "estate," "grain," and "merchandise." Such words began to drop in popularity about 1880, but they were often included after that date.

Another type of word falling in this category included terms referring to things or activities in use or in vogue during a certain period, such as "carpeting," "oilcloth," and "carriage." All of these words were used much more frequently between 1880 and 1910 than during any other 30-year period, the decrease in the last period being so distinct as to show a definite tendency for the terms to be dropped.

The third principal type of word to appear only during a certain period and then to drop out again included terms used in certain business dealings which were not found in the first of these textbooks but were added later, and now have dropped out again because they deal with phases of business with which the elementary school child never comes in contact. Chief among these words were those concerning stocks and bonds, with such related terms as "speculator," "agent," and "broker."

It seems the trends of the times are more evident through a study of the subjects for problems as shown by the concrete nouns used than through any other phase of the study of textbooks. Changes in the life of the day show up clearly and differences in the use of certain terms are also evident, but perhaps the greatest factor influencing problem subjects was brought about through changes in the purposes of textbooks. It could not be said that subject matter indicates that modern arithmetics use more practical problem subjects than did old textbooks, for apparently the vocabulary has been in keeping with the times throughout the books. However, the changed viewpoint of textbook writers, from that of making the subject all-important to that of placing the emphasis on the child, has brought about considerable change in both subject matter and terminology.

Chapter VII
Summary

In general, the picture of arithmetic textbooks—their physical make-up, content, and methods—has changed gradually but steadily through the years toward the accomplishment of a definite goal, that of giving the child the arithmetic information needed by the average individual and presenting it in such a way as not only to make it interesting and usable in his in-school and out-of-school life but also to make him recognize the need for it in his everyday social experiences.

Since textbooks of 150 years ago were written for adults rather than for children, and were primarily designed to fill a need for information about business practices, they did not contain elementary, everyday arithmetic. Their sole purpose seems to have been to provide a form of reference book to which businessmen or apprentices could turn for any mathematical information they needed.

The inclusion of arithmetic in the public school course of study, early in the nineteenth century, created a need for texts for the young child who did not yet have a foundation in the fundamentals. Colburn (1821) was the first to write an arithmetic of this kind, but the need for it was so thoroughly recognized that other writers immediately followed his lead, and from this time on almost every author stated in his preface that one of his aims had been to simplify his text so that it could be understood by the beginner in arithmetic.

With the tremendous expansion of industry and the resultant increased trade in the middle 1800's came the need for everyone, not just the businessman, to have an understanding of arithmetic in order to carry on his daily activities. Consequently the subject was given a place of importance in the course of study. By this time many states had passed compulsory education laws, and, while not altogether effective, such laws did create a much more urgent need for textbooks and teachers in the field of arithmetic. No longer was the subject taught only to those who were interested in it. Teachers and textbook writers alike began to realize that arithmetic, a hard subject for many of the children, and one for which they could see little immediate use, would have to be presented as attractively as possible. As a result great improvement took place both in the general appearance of the texts and in the techniques and devices used in presenting material.

The purposes for which textbooks in arithmetic have been written have of course kept pace with the needs of those for whom they were designed. All authors of arithmetics published a century and a half ago agreed that their primary concern was that of the businessman's needs. Sometimes books were planned for private use, sometimes for use with the aid of a tutor, but almost never were they written for the classroom. Later they were written almost entirely for schools and their purposes have changed to more nearly meet the needs of those for whom they are designed.

Whenever certain theories of teaching were in favor, material in texts has been presented in harmony with those ideas, and among the purposes expressed was that of securing outcomes expected when such theories were put into practice. Many different methods of teaching and of presenting material have been accepted at one time or another through the years, always with the purpose of securing outcomes believed to result from the use of such methods. While some of these have been proved unsound and have been dropped, their better characteristics have often remained, until today texts contain the best of what has gone before. Changes have been gradual and sometimes not permanent, but their ultimate aim has always been to teach the arithmetic needed. In recent years this objective has expanded to include the arithmetic the child needs now as well as later, and an effort has been made to help him realize that a knowledge of numerical concepts is essential to an understanding of many of the situations that will arise in his everyday life.

The physical make-up of the texts has kept abreast with changes in the needs of those for whom the books have been designed. In the early days, books were plain in appearance on the outside, difficult to read from the point of view of size of type and crowding of material, and technical and advanced in vocabulary.

At first available materials had a great deal to do with the outside appearance of texts, as leather was used for covers for almost all books published. About 1820, however, paper-covered cardboard became popular and continued to be the most common kind of cover used until toward the end of the century, when cloth-covered cardboard came into vogue. Since that time the cloth covers have been almost universally accepted. Early covers were plain, as there was no way to print them without great expense. With the introduction of paper-covered cardboard, however, the use of print on covers became very popular. Since that time pictures have been an added attraction and have gradually crowded out much of the printed material. Recent texts have had comparatively plain covers, with often a simple picture of children at play.

Although early texts were very small, sometimes not more than 4 by 6 inches, the size of the present-day arithmetic had become almost standardized as early as 1850. Noticeable improvement in the size of print and in the jumbled appearance of the page was also evident by this time. When an effort was made to present material more nearly on the level of the child for whom the book was designed, there came a division of material which led to the two-book series, and later to the further breaking down of subject matter to the level of each grade, separately. In this way specific aims and objectives of arithmetic teaching in each grade might be met. The adding of illustrations, both for instructional purposes and for their human interest value, and the use of color in these illustrations have brightened texts until today they come to children readable and full of life and fun.

On the whole, the content of arithmetic textbooks during the 150 years included in this study has not changed a great deal. Throughout

that period of time there has been certain core content that has been included in all or almost all the texts studied, with much of this core material having been handed down through the ages as vital to the study of arithmetic. Very little content has been dropped out of texts except for a few of the more complicated phases of the subject, which have been advanced to higher mathematics, and a very few phases that have become outmoded. On the other hand, much material has been added, either because its use has been introduced into society since the early years of this study or because of recent attempts to socialize arithmetic and fit the study into the problems that arise in everyday life.

The principal phases of arithmetic—the four fundamental processes, whole numbers, fractions, decimals, numeration, percentage, powers and roots, denominate numbers, measures, and ratio and proportion—were found in almost every text studied. In early books the Rule of Three was the high point of arithmetic learning, and most texts did not include material beyond that point.

Important business practices, such as interest, profit and loss, commission, insurance, discount, and U.S. currency, have also been included in almost all of the texts. There has been considerable difference of opinion among textbook writers, however, as to how much emphasis should be given to less common types of business transactions. Obsolete practices, such as barter, tare and tret, and fellowship, have of course been dropped. In their place have come topics more favorable to social usage, such as making change, handling personal checking and savings accounts, sending mail, and so forth.

The order in which the various phases of arithmetic and business practice were introduced in the texts has not varied much through the years. Whole numbers were always introduced first, with fractions and decimals almost always following in that order. Of the four fundamentals addition was always introduced first, usually followed by subtraction, multiplication, and division, respectively. In general, there was little disagreement of any consequence even in decisions as to which business practices to include first.

Trends in the forms used in presenting material have taken a steady course toward making arithmetic more practical and more interesting. In recent texts a much larger proportion of the book has been given to written and drill problems; rules, definitions, and examples have been given less space and have been presented for the pupil to use rather than to memorize; tables have dealt with more commonplace matters and fewer technical subjects; illustrations have been added to help interest the pupil and assist him in understanding arithmetic; and subject matter has been introduced in units with a central theme to help make the pupil recognize the relationship of arithmetic to ordinary situations and common experiences.

New forms of presentation have been added, too, although none of the old ones have been dropped. Illustrations to make the subject both clear and attractive, instructional problems to correlate arithmetic with other subjects in the curriculum and with out-of-school activities, and tests and drills to help the pupil to know whether he

is understanding the work presented have all been added to texts and have gained favor rapidly.

There has been a distinct tendency in recent years to include a greater variety of topics in arithmetics and to present them in more different ways. The increase in number of topics has been almost entirely in the field of daily-life activities, and in the presentation of material it has been in the field of attractive interest-catching methods. In fact, there is evidence in every phase of textbooks that authors are attempting to make pupils enjoy their work in arithmetic, to feel a need for it, and to desire to study the subject.

More change has taken place in the methods of teaching and the point of view from which textbooks are written than in any other phase of textbook development. As has been said, the first few years covered by this analysis fell in what was known as the "ciphering book" period, when pupils merely copied what the teacher set up for that purpose. In treating a topic, only four elements were recognized —the definition, the rule, the explanation, and problems falling under that rule. No effort was made to explain the material—in fact, in many cases teachers did not understand it themselves—nor was there any emphasis on either drill or speed. The purpose seemed to be purely that of giving practice in solving the type of problems met in business dealings.

Until 1821 arithmetics were written on a strictly deductive plan, but Colburn published an arithmetic in that year which caused a complete "about face" in the teaching of the subject. His inductive method was new to the field and was carried out so thoroughly and so successfully that other textbook writers immediately recognized its value and adopted it. By this method the pupil was led from the known to the unknown, and topics were introduced in their relationship to principles already studied rather than to new rules to be memorized. Although other methods have been popular at various times since Colburn's day, they have all kept this characteristic of Colburn's inductive method. Even in the activity method, which is in common use today and is a combination of the best factors in a number of methods used in the past, the inductive method is still used in the introduction of new material on all levels.

Colburn was the leader in making a number of other improvements in textbooks, all of which changes have enjoyed long periods of popularity, and many of which are still being considered best practice in arithmetics today. He was the first to go back to the object method, a method which had been used two or three centuries earlier but which had been looked down upon during the years when mathematics was taught very formally. Teaching by means of objects has an element of naturalness about it which helps the pupil develop a "feel" for numbers. This method has gradually developed into the use of entire situations rather than individual objects, and has been so successful that it is still looked upon as a very desirable means of teaching numerical relationships.

Colburn introduced "mental" arithmetic, an idea that was the forerunner of the drill materials in texts today. For half a cen-

tury almost all texts included problems grouped according to whether they were to be solved on paper or without the aid of pencil and paper. This plan, in its stricter sense, has now been discarded by most arithmetics.

Colburn was the first, also, to write an arithmetic designed especially for children. In doing this he used a vocabulary understandable to the young child, a practice copied by all of his followers, though at first not always as successfully as it might have been. In more recent years the vocabulary of textbooks has been considered of great importance and scientific studies have been conducted to determine the grade level on which certain words should first be introduced.

This characteristic of Colburn's text called to the attention of educators the wisdom of simplifying arithmetics so that they could be understood by beginners. The first evidence of an attempt toward simplification was that of omission of the more involved and less frequently used phases of arithmetic. Rules and definitions were stated in simpler form, tables were shortened or omitted altogether, wording was made less complex, and an attempt was made to arrange the introduction of new material at the ability and interest level of the child.

Provision for individual differences and the attempt to make arithmetics have a personal appeal have been among the most recent additions to textbook development. The first of these has come as a result of the testing movement, a movement that has gained rapidly in popularity in the last 20 years. Several textbook writers have provided diagnostic tests and drills, followed by remedial material, and some have also furnished "extra" work for the superior student. In this way the textbooks have become more useful to the teacher in assisting pupils individually.

The personal element in textbooks is probably the newest major development. In reality, it is a new approach to the subject, that of meeting the pupil on his own level, and of making him feel that his success in learning his arithmetic is of personal interest to all concerned. In other words, he is given the feeling that everyone is working together with a common interest, rather than that the teacher or textbook is on one side of the fence and he on the other. This point of view is doing much toward creating greater enthusiasm on the part of the pupil, because he now "belongs" instead of being an outsider who is being shown the path and told to walk it.

These new methods have of course been great motivating factors in arithmetic. In the early days only rivalry and the desire for approval were used as means of encouraging pupils to study their arithmetic. Later, various methods were used for creating the interest of the pupil in his work. The most recent form of motivation, however, has been that of developing in the pupil a feeling of need for arithmetic by showing him the many situations in which he must use such knowledge.

Changes in the form in which written problems have been presented have followed very closely those of changes in content and

method. In early texts, business transactions were by far the most prominent subjects for problems, while today everyday experiences are most often used. Although the transition from one to the other has been gradual, it has been very noticeable for more than a century.

Problems in early texts were formal, often very abstract, almost always far removed from the child's experiences, and usually stated in complicated and technical language. Typical of such problems are: "What number multiplied by 57 will produce what 134 multiplied by 71 will do?" (8:211), and "A General disposing his Army into a Square Battle, finds he has 23716 Men; Required the Number in Rank and File?" (4:288)

Large numbers were almost always involved in the problems—in teaching the fundamental processes, the handling of units as such was omitted entirely, the first problems consisting of four or five digits in each number. Written problems carried out this same policy, dealing with large quantities of goods, important business transactions, etc.

Puzzle problems, a sort of intelligence test as well as a test in arithmetic, were included in almost all texts. Sometimes they were in verse, sometimes unnecessary information was given to distract attention from the real issue, and sometimes a combination of two or more entirely separate problems was presented as a single problem.

Efforts at simplifying texts made a great difference in the wording of problems; grade leveling affected the complexity and subject matter; correlation with other subjects and with life situations brought about the grouping of problems around a central theme; and the introduction of the personal element into arithmetics changed the point of view from which problems were written. The transition, from the uninteresting and impersonal problems in early texts to such social, personal problems as "The *health* book says to drink six glasses of water a day. I drank 1 at breakfast, 2 in the morning, 2 at lunch, and 2 at supper. Did I drink enough?" (57b:30) found in present-day arithmetics, has been one of the most important developments in the textbook field. On the whole, questions in modern texts are not only practical, they are interesting, informational, and socially useful.

In every phase of the textbooks there is evidence of the fact that authors are realizing more and more that in school work, as well as at home, children learn more quickly and with greater enthusiasm if they see the need for it and have it presented to them in an attractive way. Texts have not only become more appealing in appearance and point of view, but have been made to fit into the social life of the child. In other words, the pupil now sees the mathematical relationships in his surroundings, and he enjoys his arithmetic because it is a part of life instead of something entirely outside the realm of his experience. The subject is no longer taught as a preparation for later life, it *is* life, with each new phase of arithmetic falling into place naturally and appearing as a new challenge.

A List of Bulletins in the Field of Education, Indiana University

By Kathleen Dugdale

The School of Education at Indiana University has published the following bulletins. All of these may be obtained from the University Bookstore for 50 cents postpaid unless otherwise noted.

Proceedings of the High School Principals' Conference (November 23 and 24, 1923). Vol. I, No. 1, 1924. 85 p. (Supply exhausted.)

Investigation of Nursing as a Professional Opportunity for Girls, Part I, Technical Study; Part II, Vocational Information Bulletin. By Florence E. Blazier. Vol. I, No. 2, 1924. 69 p.

Proceedings of the Eleventh Conference on Educational Measurements. Vol. I, No. 3, 1925. 141 p.

Proceedings of the High School Principals' Conference (November 14 and 15, 1924). Vol. I, No. 4, 1925. 49 p. (Supply exhausted.)

First Revision of the Bibliography of Educational Measurements. Compiled by the Bureau of Cooperative Research. Vol. I, No. 5, 1925. 147 p. (Supply exhausted.)

Proceedings of the Twelfth Conference on Educational Measurements. Vol. I, No. 6, 1925. 76 p.

The Effect of Population upon Ability to Support Education. By Harold F. Clark. Vol. II, No. 1, 1925. 28 p.

Proceedings of the High School Principals' Conference (November 20 and 21, 1925). Vol. II, No. 2, 1925. 77 p. (Supply exhausted.)

A Cross-Indexed Bibliography on School Budgets. By Harold F. Clark. Vol. II, No. 3, 1926. 66 p.

A Comparison of the Results Made on Certain Standardized Tests by Pupils in the Bloomington High School Who Were Taught in Classes of the Same Grade by University Student Teachers and by Regular High School Teachers. By Carl G. F. Franzén. Vol. II, No. 4, 1926. 19 p.

Proceedings of the Thirteenth Annual Conference on Educational Measurements. Vol. II, No. 5, 1926. 103 p.

When to Issue School Bonds. By Harold Florian Clark and Paul Royalty. Vol. II, No. 6, 1926. 16 p.

Students' Attitude Toward Examinations. By Grover T. Somers. Vol. III, No. 1, 1926. 48 p.

Proceedings of the High School Principals' Conference (November 12 and 13, 1926). Vol. III, No. 2, 1926. 27 p.

Index Numbers in School Administration. By Harold F. Clark. Vol. III, No. 3, 1927. 35 p.

Topical Analysis of 234 School Surveys. Compiled by the Bureau of Cooperative Research. Vol. III, No. 4, 1927. 111 p. (Supply exhausted.)

Proceedings of the Fourth Annual Conference on Elementary Supervision. Vol. III, No. 5, 1927. 64 p.

Proceedings of the Fourteenth Annual Conference on Educational Measurements. Vol. III, No. 6, 1927. 66 p.

Some Phases of the Junior College Movement. By I. Owen Foster, Harold F. Clark, Willard W. Patty, and Leo M. Chamberlain. Vol. IV, No. 1, 1927. 125 p. (Supply exhausted.)

Second Revision of the Bibliography of Educational Measurements. By Henry Lester Smith and Wendell William Wright. Vol. IV, No. 2, 1927. 251 p. (75¢)

Bibliography of School Buildings, Grounds, and Equipment, Part I. By Henry Lester Smith and Leo Martin Chamberlain. Vol. IV, No. 3, 1928. 326 p. (75¢)

Proceedings of the High School Principals' Conference (November 18 and 19, 1927). Vol. IV, No. 4, 1928. 54 p.

The Economic Effects of Education. By Harold F. Clark. Vol. IV, No. 5, 1928. 39 p.

Proceedings of the Fifteenth Annual Conference on Educational Measurements. Vol. IV, No. 6, 1928. 73 p.

Proceedings of the Fifth Annual Conference on Elementary Supervision. Vol. V, No. 1, 1928. 54 p.

Proceedings of the High School Principals' Conference (November 16 and 17, 1928). Vol. V, No. 2, 1928. 33 p.

The Development and Use of a Composite Achievement Test. By Wendell William Wright. Vol. V, No. 3, 1929. 90 p.

An Analysis of the Attitudes of American Educators and Others Toward a Program of Education for World Friendship and Understanding. By Henry Lester Smith and Leo Martin Chamberlain. Vol. V, No. 4, 1929. 109 p.

Tentative Program for Teaching World Friendship and Understanding in Teacher Training Institutions and in Public Schools for Children Who Range from Six to Fourteen Years of Age. By Henry Lester Smith and Sherman Gideon Crayton. Vol. V, No. 5, 1929. 54 p.

Proceedings of the Sixteenth Annual Conference on Educational Measurements. Vol. V, No. 6, 1929. 96 p.

Proceedings of the Sixth Annual Conference on Elementary Supervision. Vol. VI, No. 1, 1929. 73 p.

An Analysis of the Duties of County School Superintendents and Superintendents of Schools in Certain Cities in Indiana. By Henry Lester Smith and Leo Martin Chamberlain. Vol. VI, No. 2, 1929. 94 p.

Proceedings of the High School Principals' Conference (November 22 and 23, 1929). Vol. VI, No. 3, 1930. 51 p.

Cooperative Studies in Secondary Education. By Henry Lester Smith and Carl G. F. Franzén. Vol. VI, No. 4, 1930. 121 p.

Proceedings of the Seventeenth Annual Conference on Educational Measurements. Vol. VI, No. 5, 1930. 103 p.

Proceedings of the Seventh Annual Conference on Elementary Supervision. Vol. VI, No. 6, 1930. 102 p.

A Study in Teacher Supply and Demand in Indiana. By I. Owen Foster, Robert K. Devricks, Harry N. Fitch, Earl C. Bowman, and George L. Roberts. Vol. VII, No. 1, 1930. 77 p.

Proceedings of the High School Principals' Conference (November 7 and 8, 1930). Vol. VII, No. 2, 1930. 70 p.

The Philosophy of Human Relations: Individual and Collective—A Source Book. By Henry Lester Smith and Harold Littell. Vol. VII, No. 3, 1931. 326 p. (75¢)

The Psychology of Human Relationships: Individual and Social—A Source Book. By Henry Lester Smith and Levi McKinley Krueger. Vol. VII, No. 4, 1931. 107 p.

Proceedings of the Eighteenth Annual Conference on Educational Measurements. Vol. VII, No. 5, 1931. 83 p.

Efficiency in College Management. By John Dale Russell. Vol. VII, No. 6, 1931. 143 p.

Bibliography of School Surveys and of References on School Surveys. By Henry Lester Smith and Edgar Alvin O'Dell. Vol. VIII, Nos. 1 and 2, 1931. 212 p. (75¢)

Proceedings of the Nineteenth Annual Conference on Educational Measurements. Vol. IX, No. 1, 1932. 139 p.

Bibliography of School Buildings, Grounds, and Equipment—Part II. By Henry Lester Smith and Forest Ruby Noffsinger. Vol. IX, No. 2, 1933. 182 p.

Bibliography of School Buildings, Grounds, and Equipment—Part III. By Henry Lester Smith and Forest Ruby Noffsinger. Vol. IX, No. 3, 1933. 130 p.

A Brief Summary of Literature on Leadership. By Henry Lester Smith and Levi McKinley Krueger. Vol. IX, No. 4, 1933. 80 p.

Proceedings of the Twentieth Annual Conference on Educational Measurements. Vol. X, No. 1, 1933. 98 p.

Bibliography of College and University Buildings, Grounds, and Equipment. By Henry Lester Smith and Forest Ruby Noffsinger. Vol. X, No. 2, 1934. 199 p. (75¢)

Factors Affecting the Organization of School Attendance Units. By Raleigh Warren Holmstedt. Vol. X, No. 3, 1934. 32 p.

The Old World Historical Background of Religious and Moral Education in Schools. By Henry Lester Smith, Robert Stewart McElhinney, and George Renwick Steele. Vol. X, No. 4, 1934. 144 p.

Proceedings of the Twenty-first Annual Conference on Educational Measurements. Vol. XI, No. 1, 1934. 96 p.

Bibliography of School Buildings, Grounds, and Equipment—Part IV. By Henry Lester Smith and Forest Ruby Noffsinger. Vol. XI, No. 2, 1935. 216 p.

A Brief Survey of Present-Day Religious and Moral Education in the Schools of Countries Other Than the United States of America. By Henry Lester Smith, Robert Stewart McElhinney, and George Renwick Steele. Vol. XI, No. 3, 1935. 185 p.

Some Practical Efforts to Teach Good Will. By Henry Lester Smith and Peyton Henry Canary. Vol. XI, No. 4, 1935. 169 p.

Proceedings of the Twenty-second Annual Conference on Educational Measurements. Vol. XII, No. 1, 1935. 97 p.

A Basis for the Improvement of Education in Rural Monroe County, Indiana. By Henry Lester Smith and Forest Ruby Noffsinger. Vol. XII, No. 2, 1936. 168 p.

Reading Readiness—A Prognostic Study. By Wendell William Wright. Vol. XII, No. 3, 1936. 46 p.

Proceedings of the Twenty-third Annual Conference on Educational Measurements. Vol. XII, No. 4, 1936. 60 p.

Bibliography of Literature on the Teaching of English. By Henry Lester Smith and William Isaac Painter. Vol. XIII, No. 1, 1936. 299 p.

Bibliography of Literature on Education in Countries Other Than the United States of America. By Henry Lester Smith and William Isaac Painter. Vol. XIII, No. 2, 1937. 341 p. (75¢)

Character Development Through Religious and Moral Education in the Public Schools of the United States. By Henry Lester Smith, Robert Stewart McElhinney, and George Renwick Steele. Vol. XIII, No. 3, 1937. 137 p.

Proceedings of the Twenty-fourth Annual Conference on Educational Measurements. Vol. XIII, No. 4, 1937. 100 p.

Bibliography of Literature on Education in Countries Other Than the United States of America (January 1, 1919, to December 31, 1924). By Henry Lester Smith and William Isaac Painter. Vol. XIV, No. 1, 1937. 139 p.

Two Studies in Education. By Henry Lester Smith and Merrill Thomas Eaton. Vol. XIV, No. 2, 1938. 42 p.

Bibliography of School Surveys and of References on School Surveys (Supplement to Vol. VIII, Nos. 1 and 2). By Henry Lester Smith and Edgar Alvin O'Dell. Vol. XIV, No. 3, 1938. 144 p. (75¢)

Proceedings of the Twenty-fifth Annual Conference on Educational Measurements. Vol. XIV, No. 4, 1938. 37 p.

Remedial Instruction in Reading with College Freshmen. By Nila Banton Smith. Vol. XV, No. 1, 1938. 35 p.

Two Studies in Arithmetic. By Henry Lester Smith and Merrill Thomas Eaton. Vol. XV, No. 2, 1939. 91 p.

The Relation of Retention to Speed of Learning. By Henry Lester Smith and Merrill T. Eaton. Vol. XV, No. 3, 1939. 105 p.

Proceedings of the Twenty-sixth Annual Conference on Educational Measurements. Vol. XV, No. 4, 1939. 53 p.

An Experimental Study of Standards for the Selection of Candidates for Teacher-Training Institutions. By Henry Lester Smith and Merrill Thomas Eaton. Vol. XVI, No. 1, 1939. 72 p.

State Control of Public School Finance. By Raleigh Warren Holmstedt. Vol. XVI, No. 2, 1940. 71 p.

Teacher Tenure in Indiana and the Courts. By Henry Lester Smith and R. Foster Scott. Vol. XVI, No. 3, 1940. 81 p.

Twenty-seventh Annual Conference on Educational Measurements. Vol. XVI, No. 4, 1940. 37 p.

A Study of the English Usage, Spelling, and Vocabulary of 251 Graduate Students at Indiana University. By Henry Lester Smith and Merrill T. Eaton. Vol. XVI, No. 5, 1940. 71 p.

The Teaching of Arithmetic to Low-Ability Students in the Elementary Schools. By Henry Lester Smith and Merrill T. Eaton. Vol. XVI, No. 6, 1940. 130 p.

An Analysis of the Achievement of 164 Sophomores Enrolled in Teacher-Training Courses at Indiana University. By Henry Lester Smith and Merrill T. Eaton. Vol. XVII, No. 1, 1941. 31 p.

The Scholastic Achievement of Athletes at Indiana University. By Henry Lester Smith and Merrill T. Eaton. Vol. XVII, No. 2, 1941. 16 p.

Twenty-eighth Annual Conference on Educational Measurements. Vol. XVII, No. 3, 1941. 42 p.

Legal Relations of Pupils and Parents to the Public Schools of Indiana—Part I. By Henry Lester Smith and R. Foster Scott. Vol. XVII, No. 4, 1941. 81 p.

Legal Relations of Pupils and Parents to the Public Schools of Indiana—Part II. By Henry Lester Smith and R. Foster Scott. Vol. XVII, No. 5, 1941. 72 p.

A Study of Students Transferring from Indiana University. By Merrill T. Eaton. Vol. XVII, No. 6, 1941. 26 p.

An Analysis of Arithmetic Textbooks (First Period—1790-1820). By Henry Lester Smith and Merrill T. Eaton. Vol. XVIII, No. 1, 1942. 55 p.

Education in Guatemala. By John Espéndez-Navarro and Henry Lester Smith. Vol. XVIII, No. 2, 1942. 96 p.

A Study of Indiana University Withdrawals. By Merrill Thomas Eaton. Vol. XVIII, No. 3, 1942. 20 p.

Education in Mexico. By Henry Lester Smith. Vol. XVIII, No. 4, 1942. 95 p.

Twenty-ninth Annual Conference on Educational Measurements, and *A Study of the Achievement of Freshman Students Holding County Scholarships.* By Merrill T. Eaton. Vol. XVIII, No. 5, 1942. 57 p.

An Analysis of Arithmetic Textbooks (Second Period—1821 to 1850, and Third Period—1851 to 1880). By Henry Lester Smith and Merrill Thomas Eaton. Vol. XVIII, No. 6, 1942. 112 p.

World War Information. By Margaret I. Rufsvold. Vol. XIX, No. 1, 1943. 129 p.

A Measure of Motor Fitness for College Men. By Karl W. Bookwalter and Carolyn W. Bookwalter. Vol. XIX, No. 2, 1943. 26 p.

A Study of Students Transferring to Indiana University from Other Colleges and Universities. By Merrill T. Eaton. Vol. XIX, No. 3, 1943. 39 p.

An Analysis of Arithmetic Textbooks (Fourth Period—1881 to 1910). By Henry Lester Smith and Merrill Thomas Eaton. Vol. XIX, No. 4, 1943. 62 p.

Further Studies of Indiana University Motor Fitness Index. By Karl W. Bookwalter. Vol. XIX, No. 5, 1943. 48 p.

An Analysis of Arithmetic Textbooks (Fifth Period—1911 to 1940). By Henry Lester Smith and Merrill Thomas Eaton Vol. XIX, No. 6, 1943. 45 p.

An Analytical Study of January-February (1943) Entering Freshmen at Indiana University. By William H. Fox. Vol. XX, No. 1, 1944. 50 p.

A Survey of the Achievement in Arithmetic of 11,348 Sixth Grade Pupils in 486 Schools in Indiana. By Merrill T. Eaton. Vol. XX, No. 2, 1944. 66 p.

A Survey of the Achievement in Social Studies of 10,220 Sixth Grade Pupils in 464 Schools in Indiana. By Merrill T. Eaton. Vol. XX, No. 3, 1944. 68 p.

Directory of Educational Research Agencies in the United States. By Henry Lester Smith. Vol. XX, No. 4, 1944. 24 p.

The Teacher Situation in Indiana. Vol. XX, No. 5, 1944. 38 p.

Planning Postwar School Buildings: Proceedings. Vol. XX, No. 6, 1944. 119 p.

One Hundred Fifty Years of Arithmetic Textbooks. By Henry Lester Smith, Merrill T. Eaton, and Kathleen Dugdale. Vol. XXI, No. 1, 1945. 153 p.